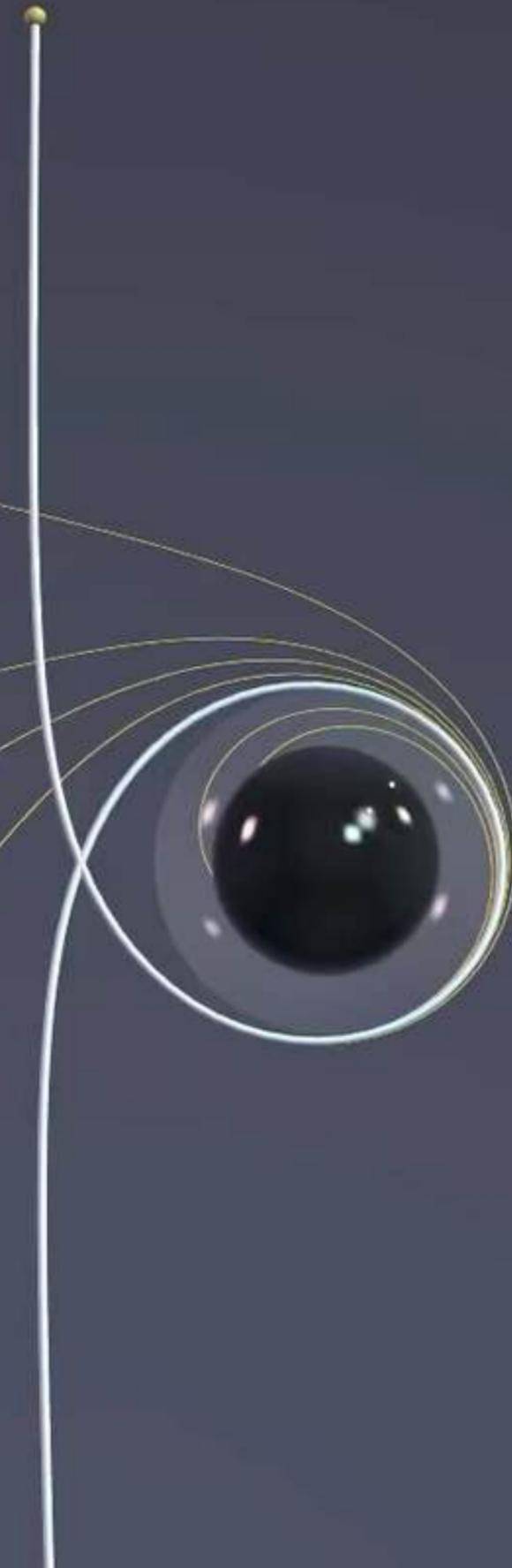


Classical Optics for Charged Black Holes

Steve Trettel,
University of San Francisco, USA



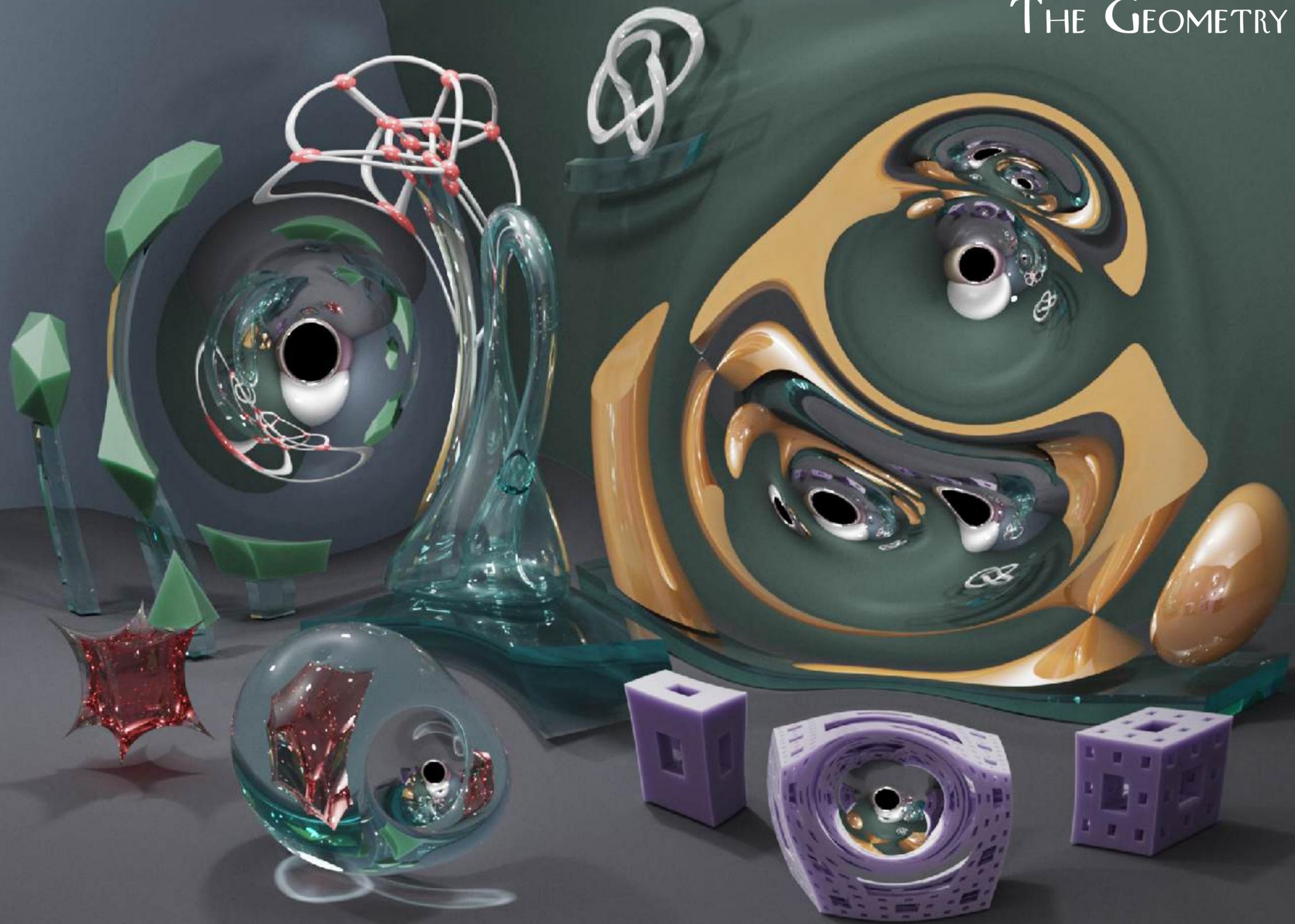
THE GEOMETRY OF SPACE(TIME)

2025

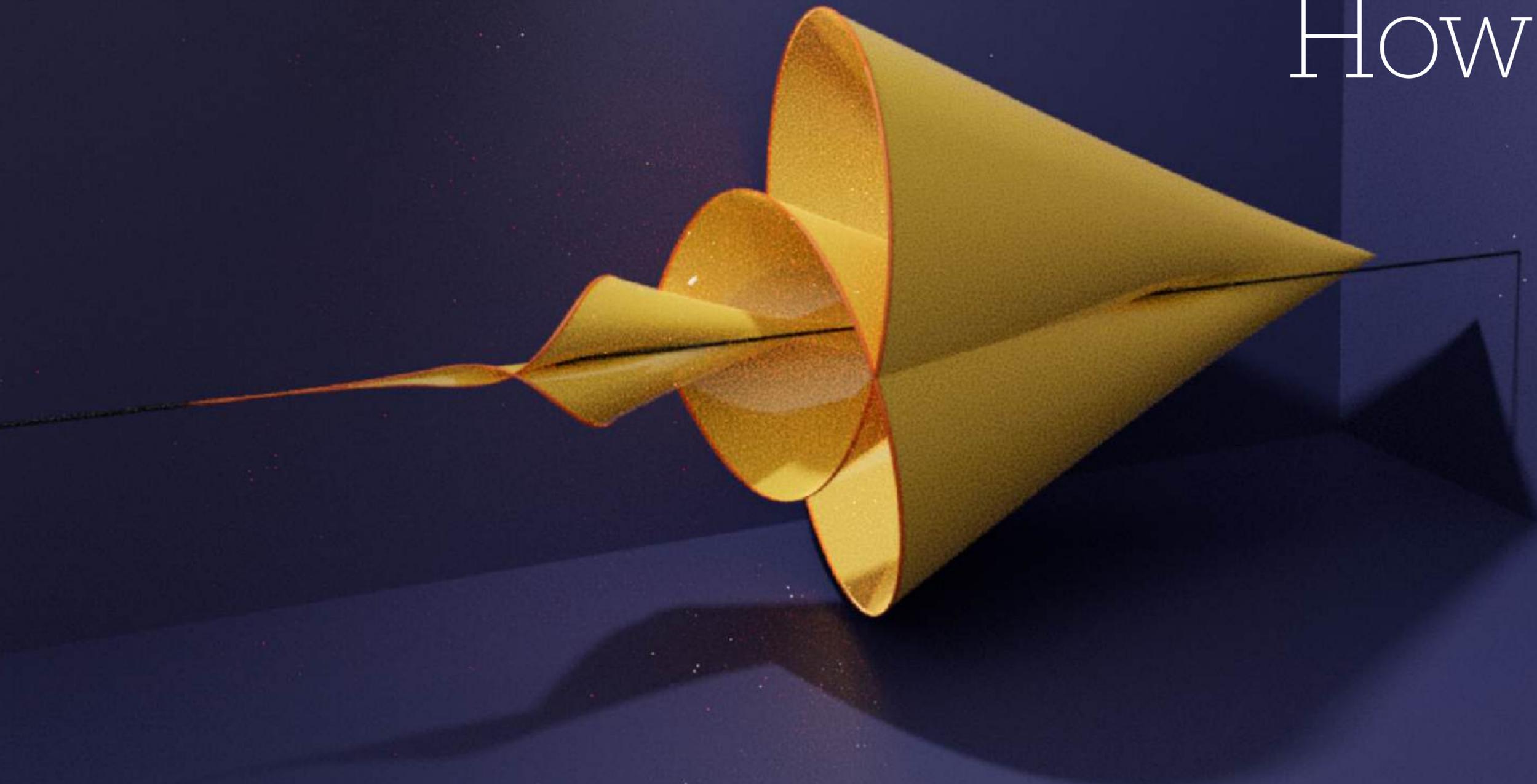


THE GEOMETRY OF SPACE(TIME)

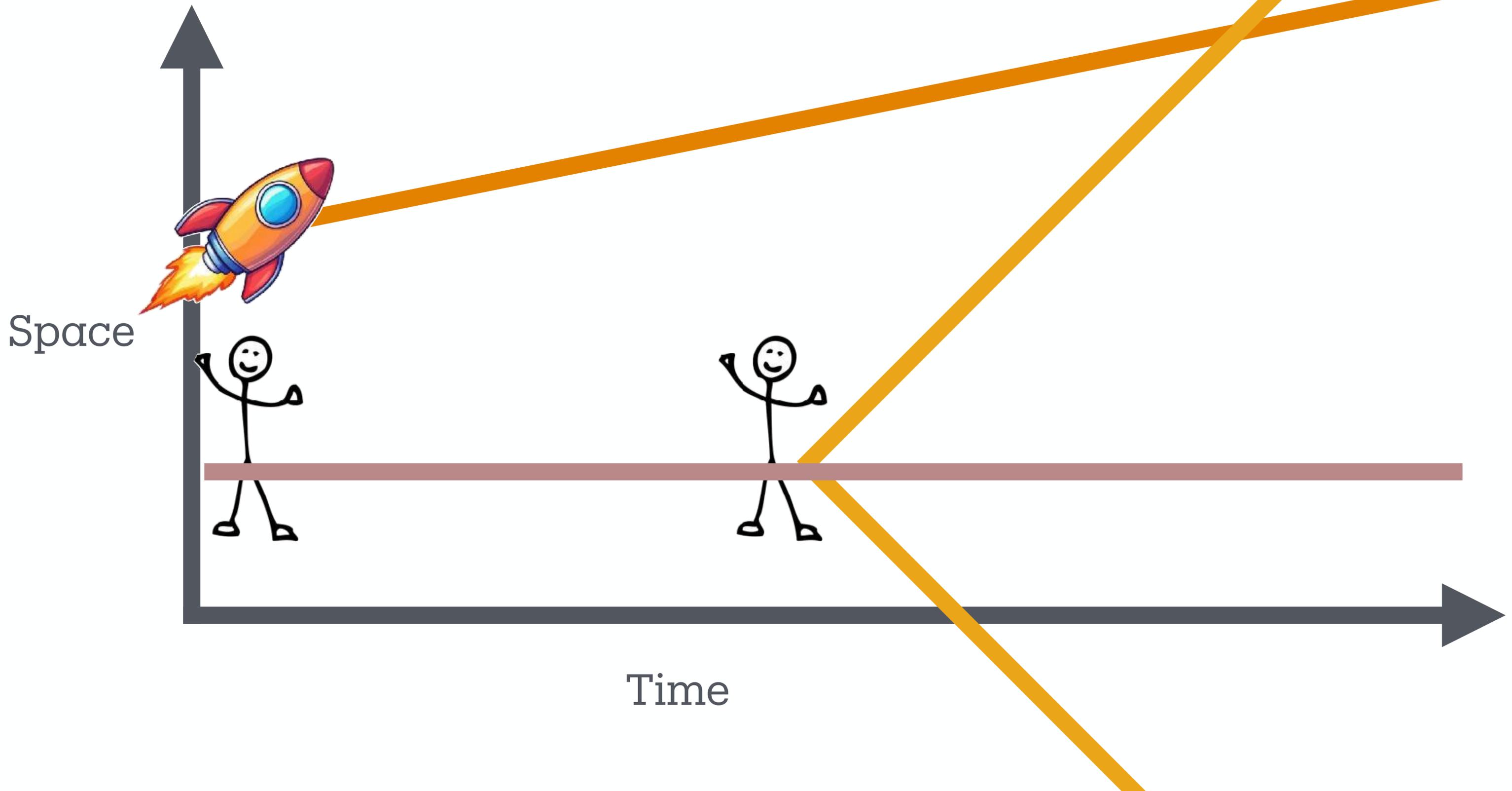
2025



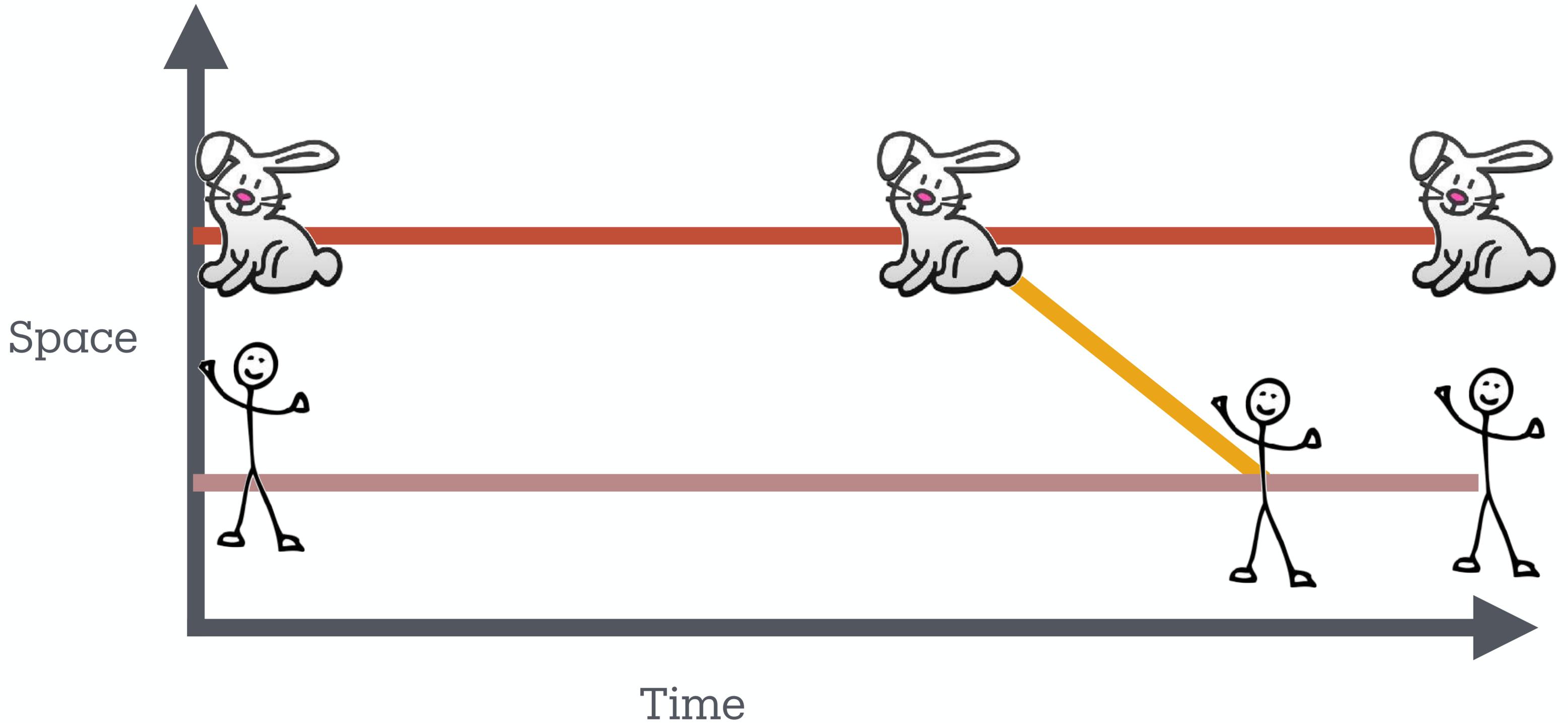
I
How to See

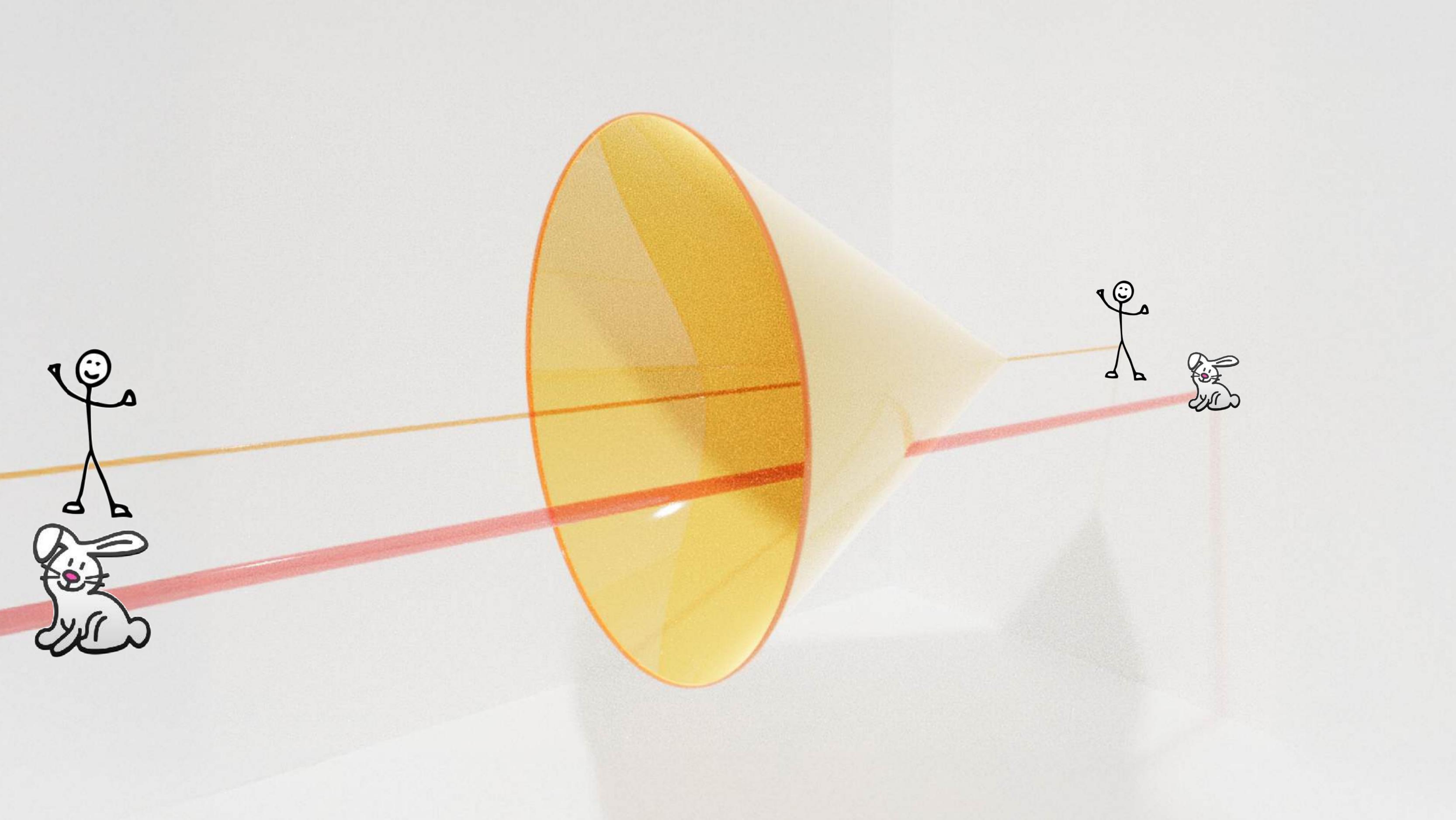


Tracing Light in Spacetime

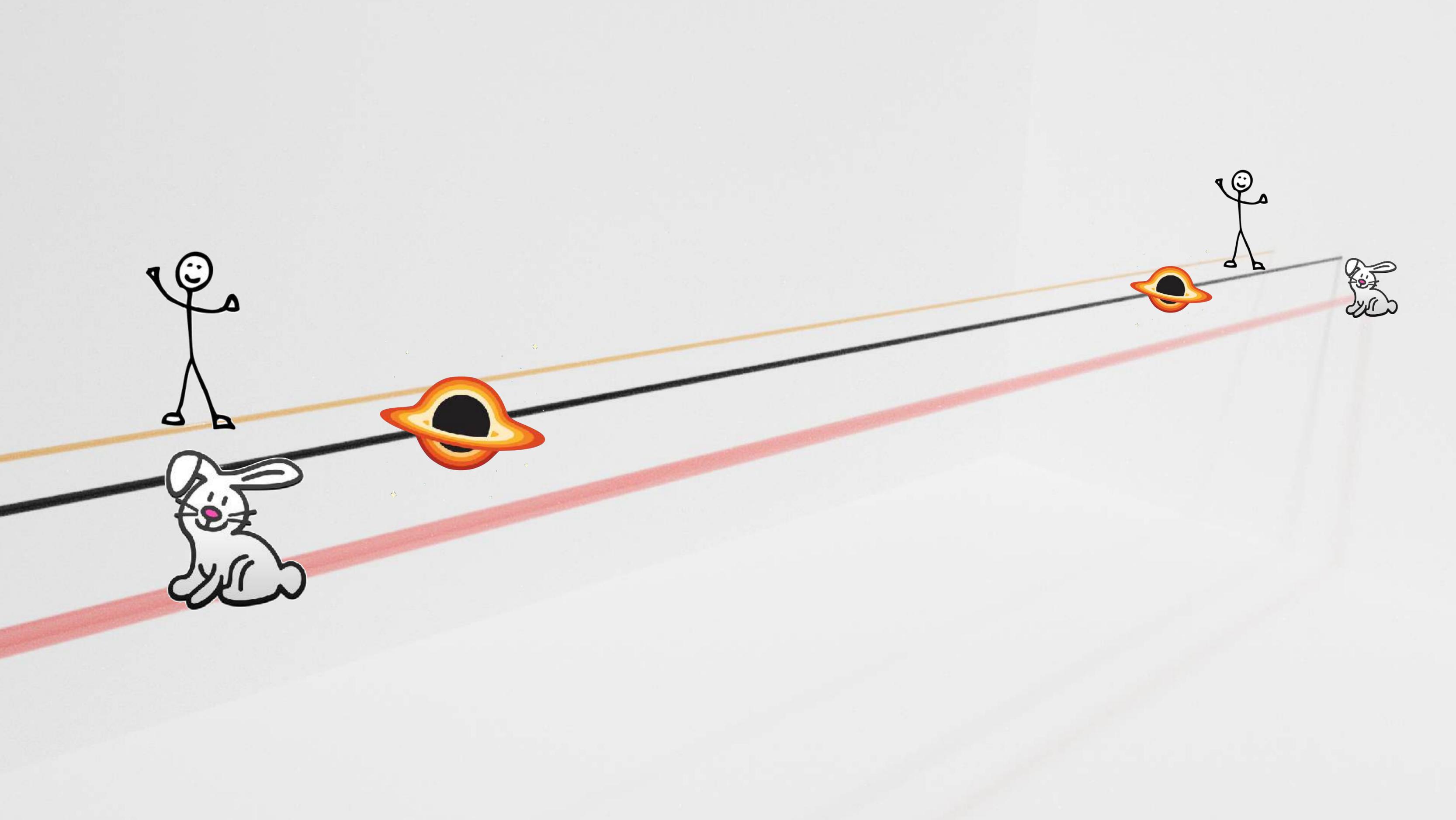


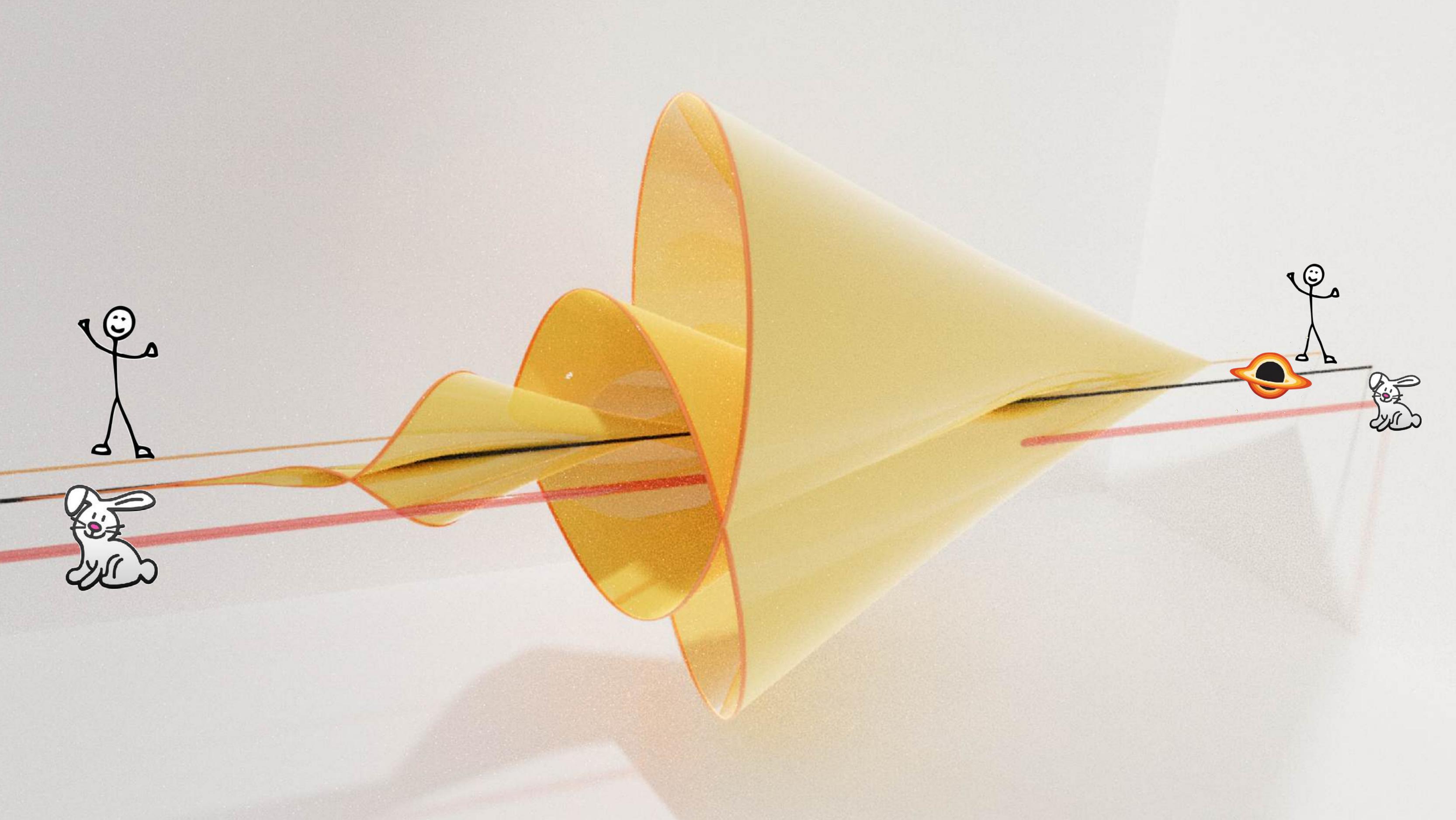
Tracing Light in Spacetime

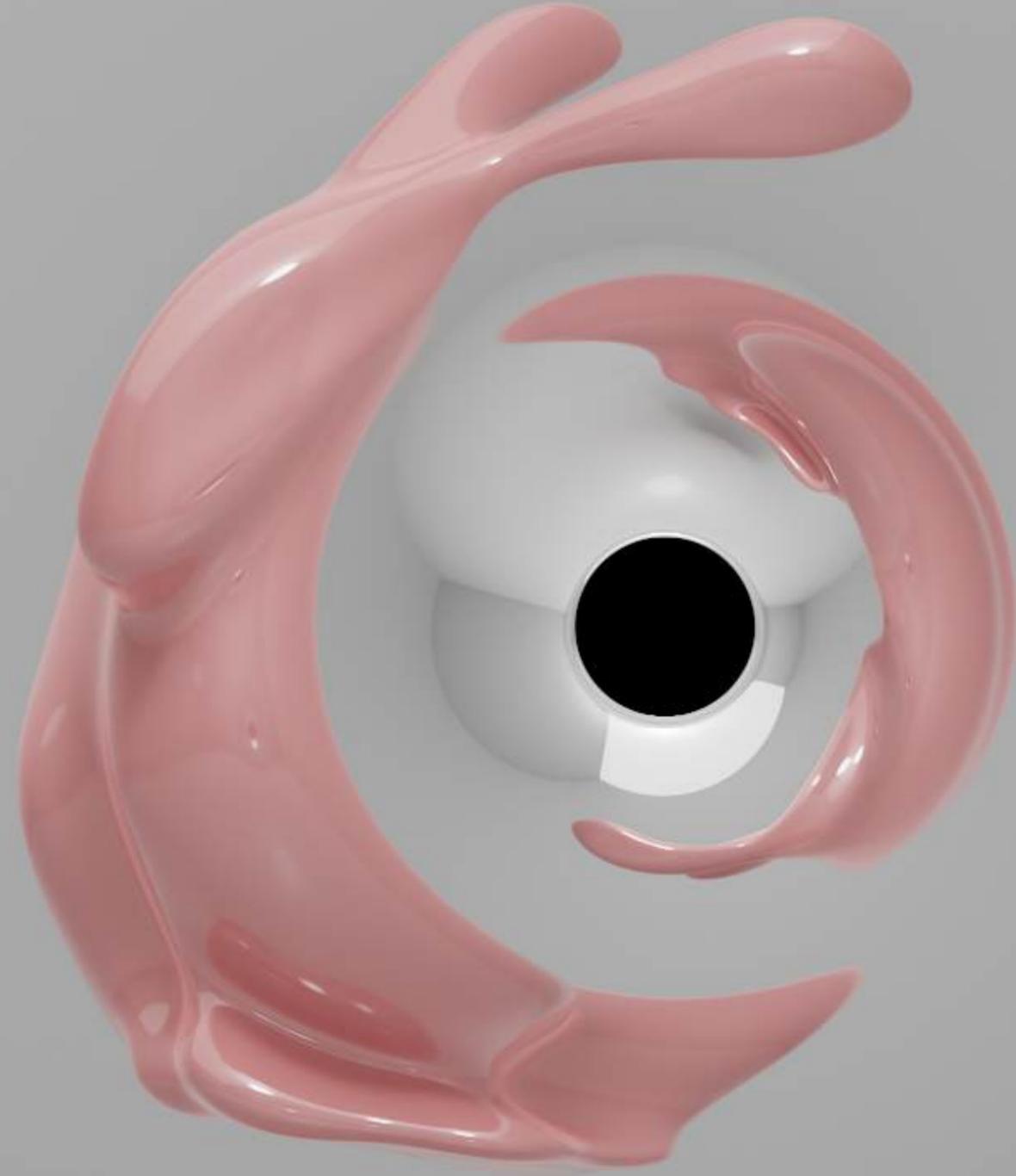






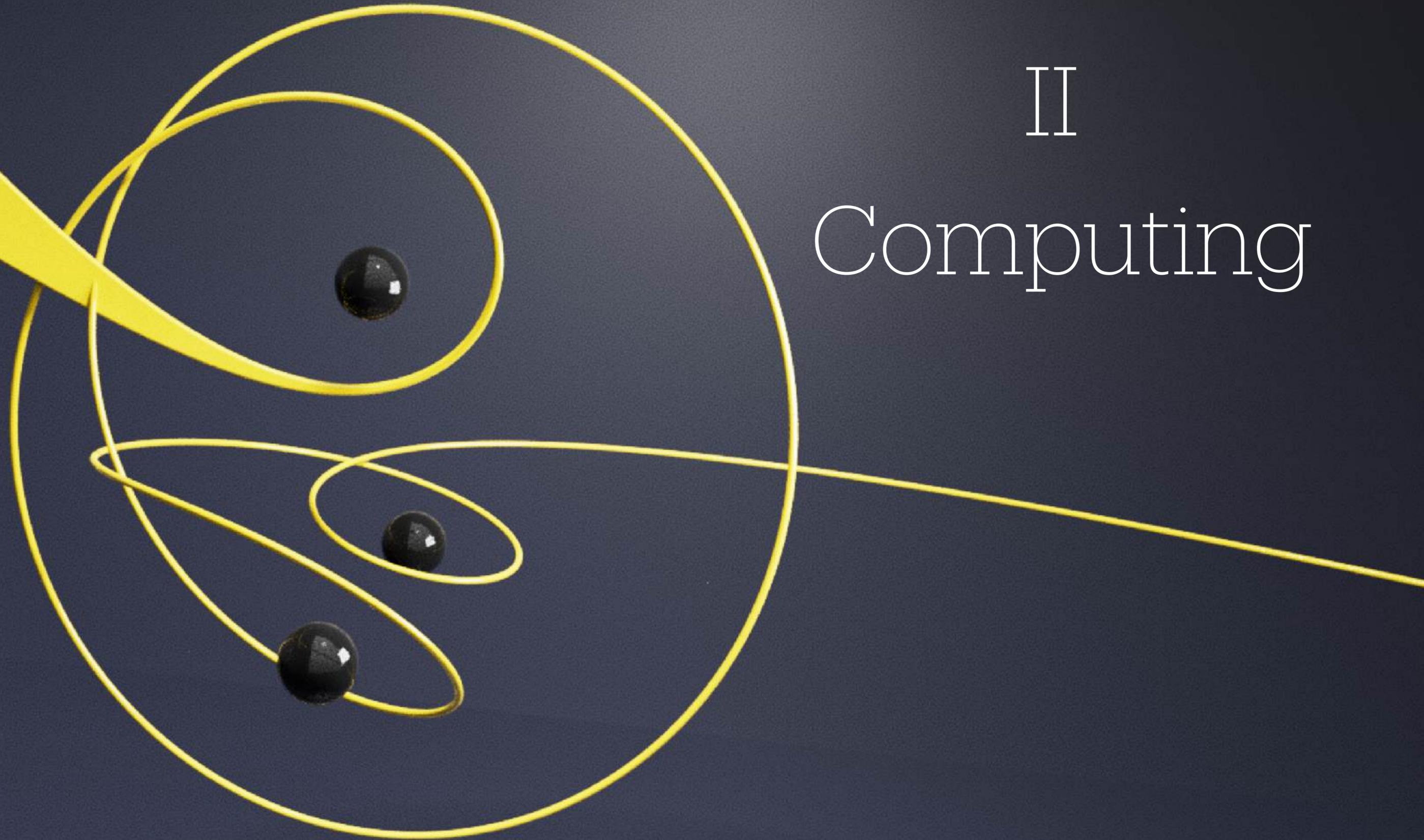






II

Computing

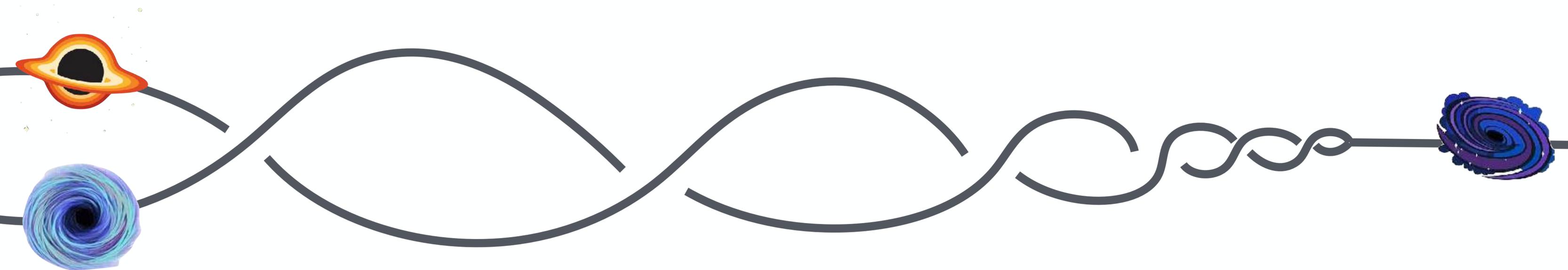




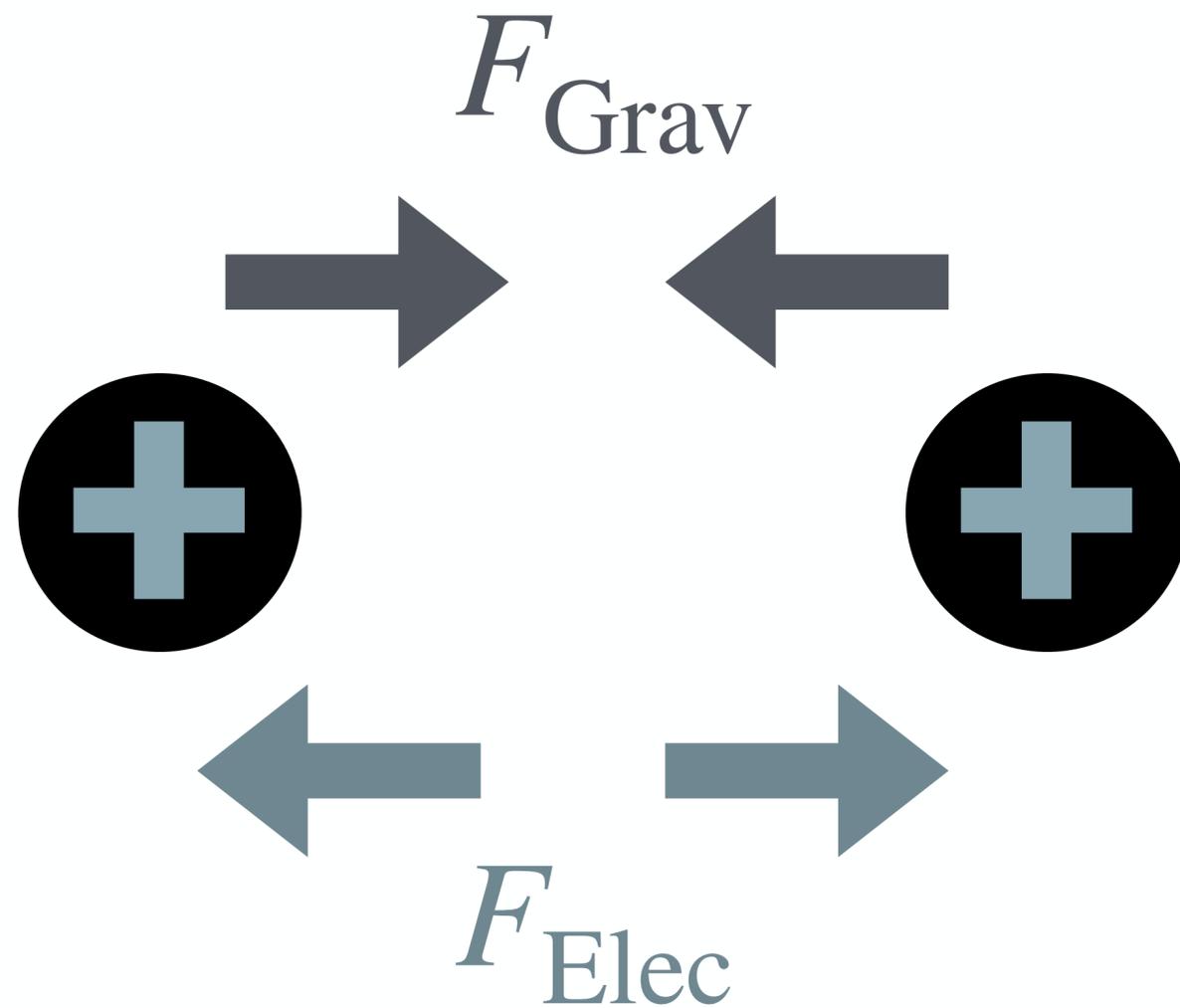
Multiple Black Holes

Generically, very difficult!
(Nonlinearity of General Relativity)

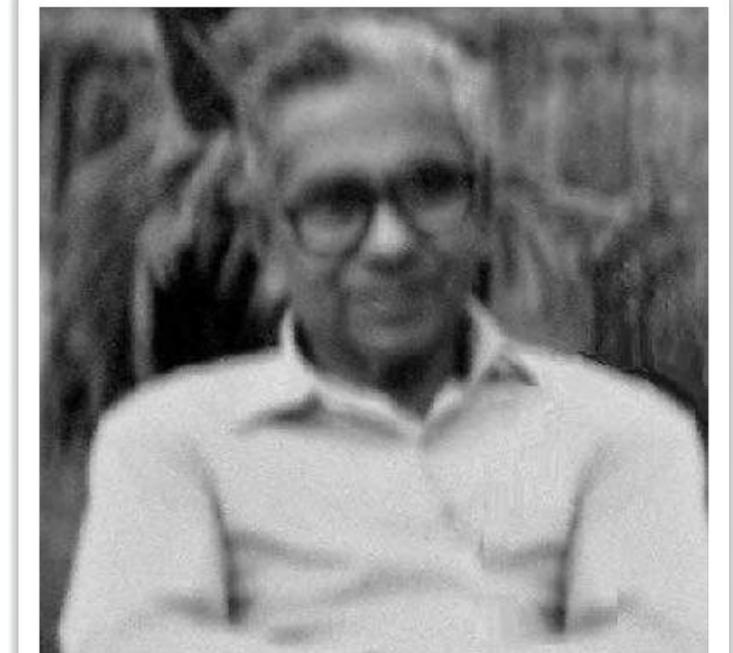
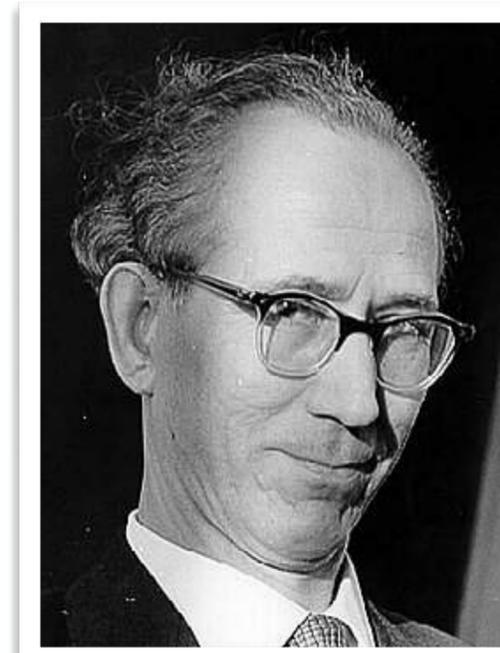
Mergers, gravitational
waves, no symmetry



Majumdar & Papapetrou (1947)



$$U = 1 + \sum_i \frac{m_i}{r_i} \quad g = -U^{-2}dt^2 + U^2ds^2$$



Discovered a solution to the Einstein-Maxwell equations containing multiple charged black holes, in electrostatic equilibrium.



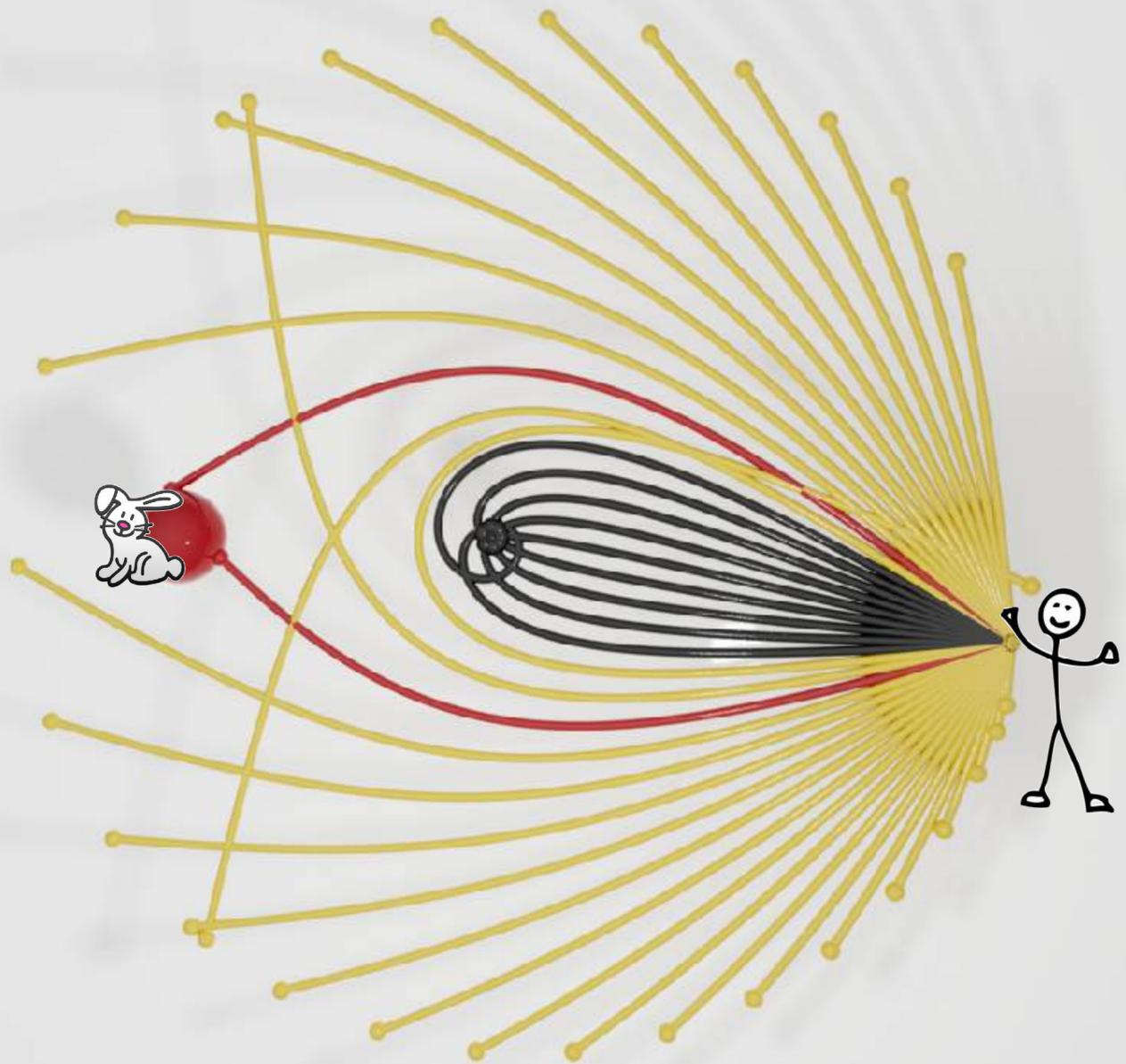


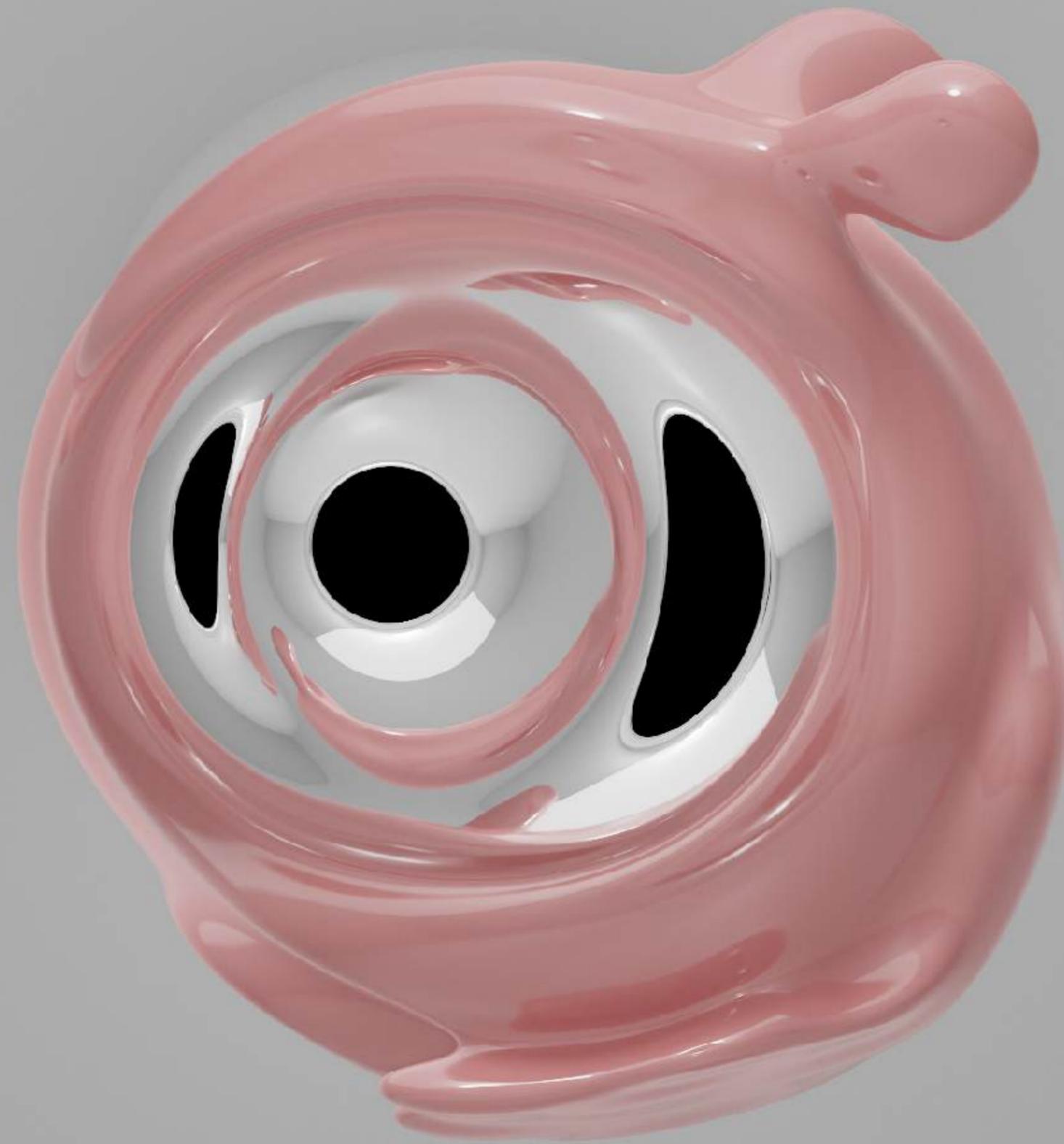
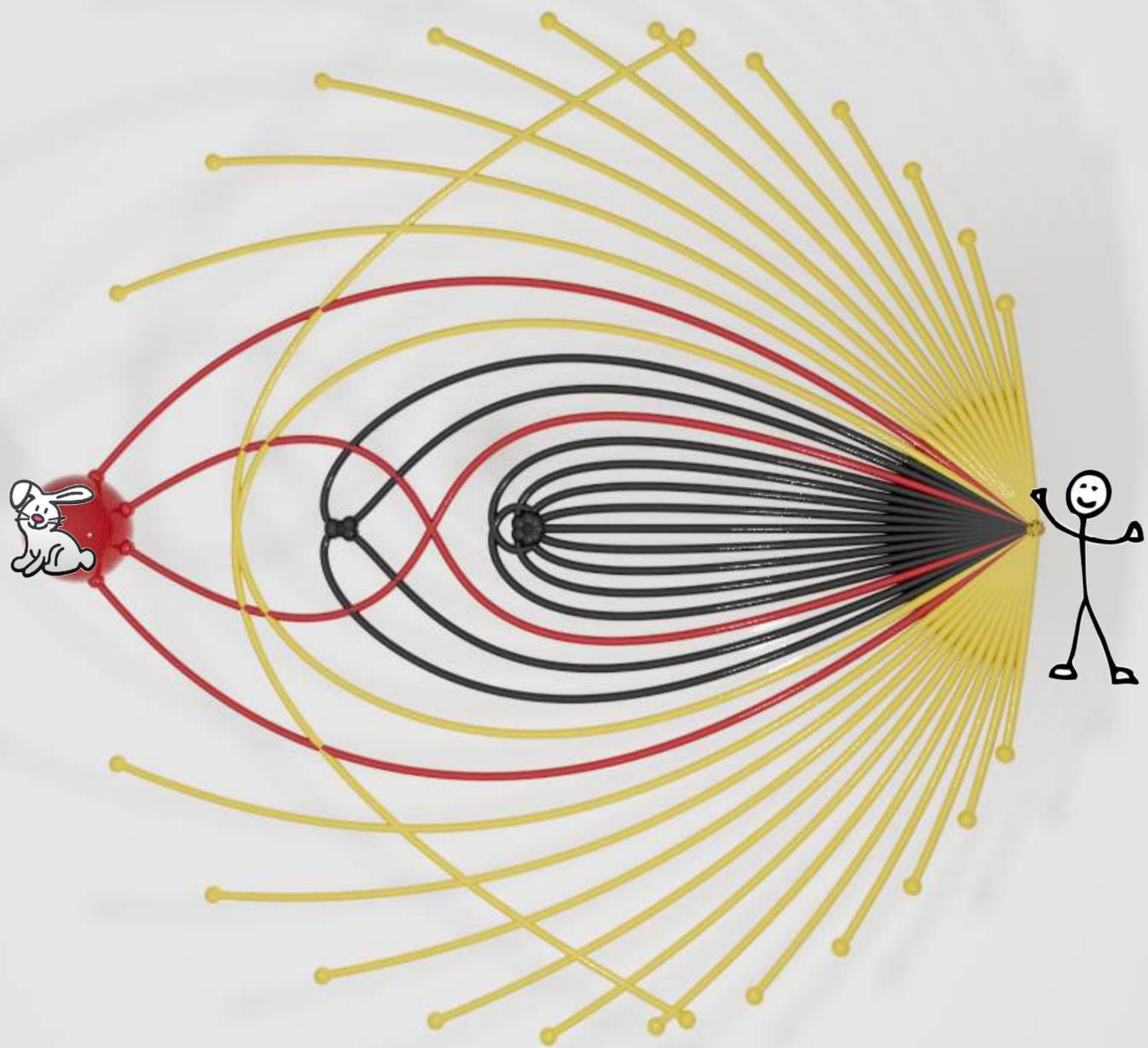
Conformal
rescaling
simplifies the
spacetime

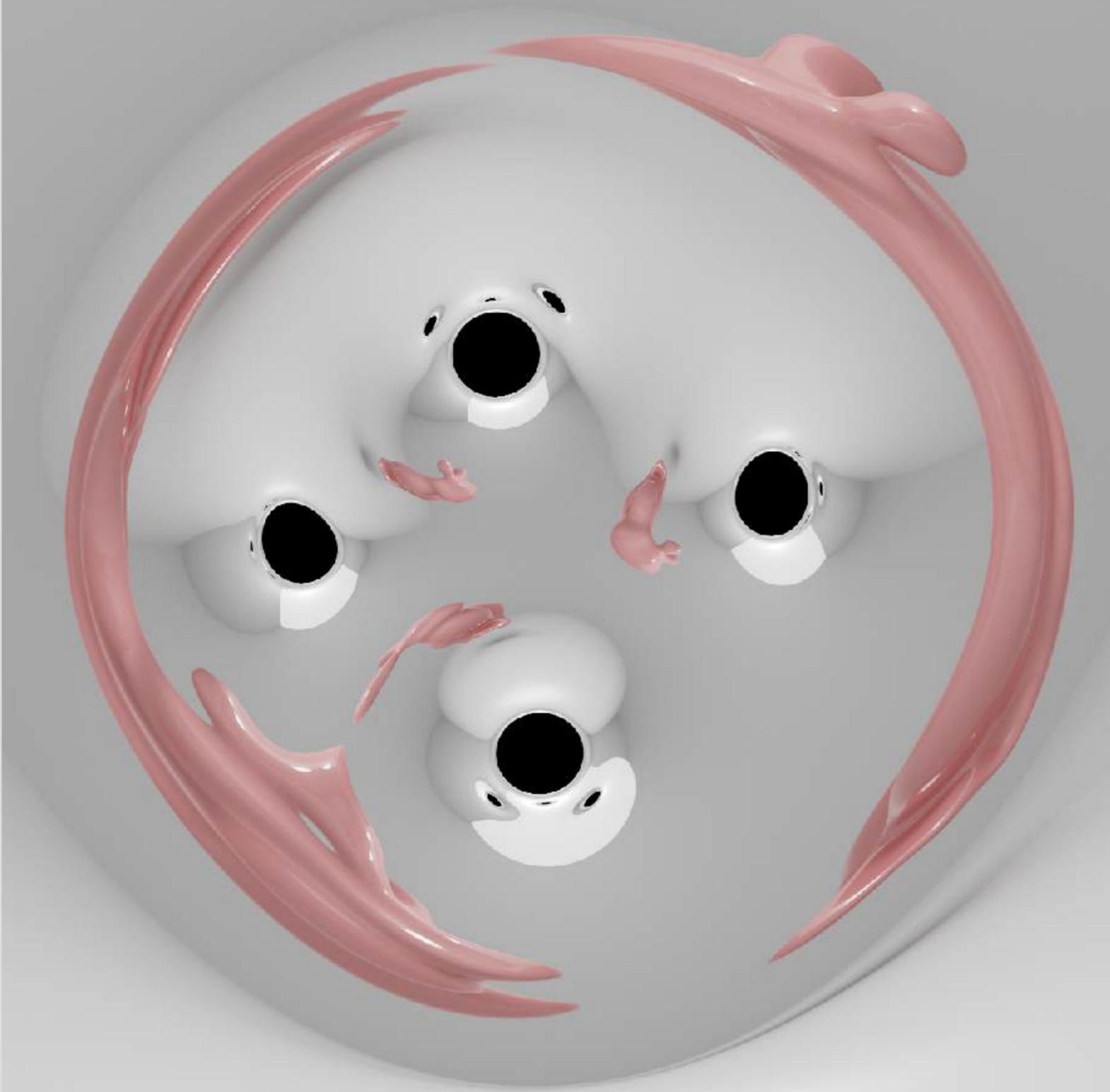
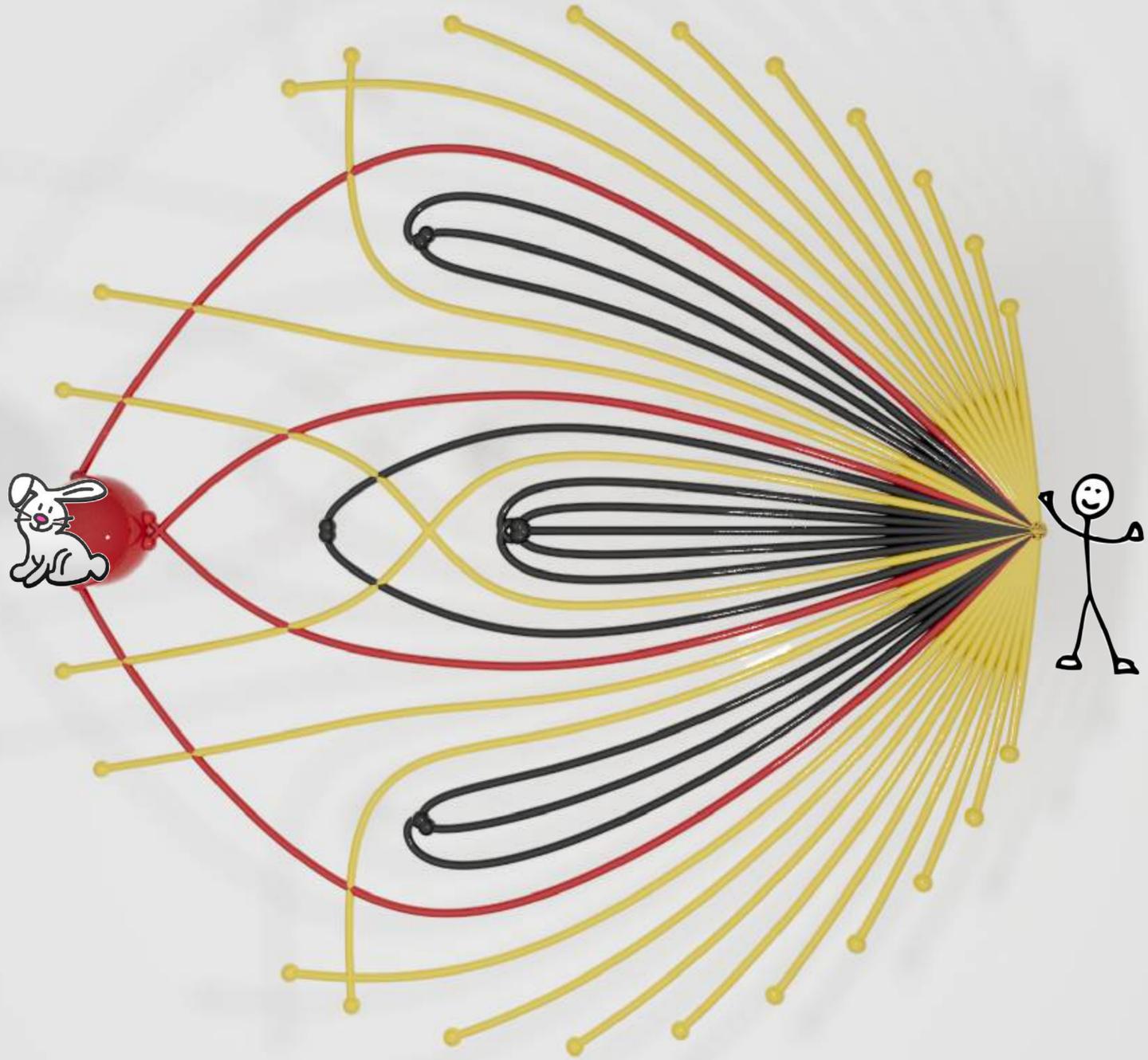


Symmetries of this solution
let us trace light's path
through space, instead of
spacetime



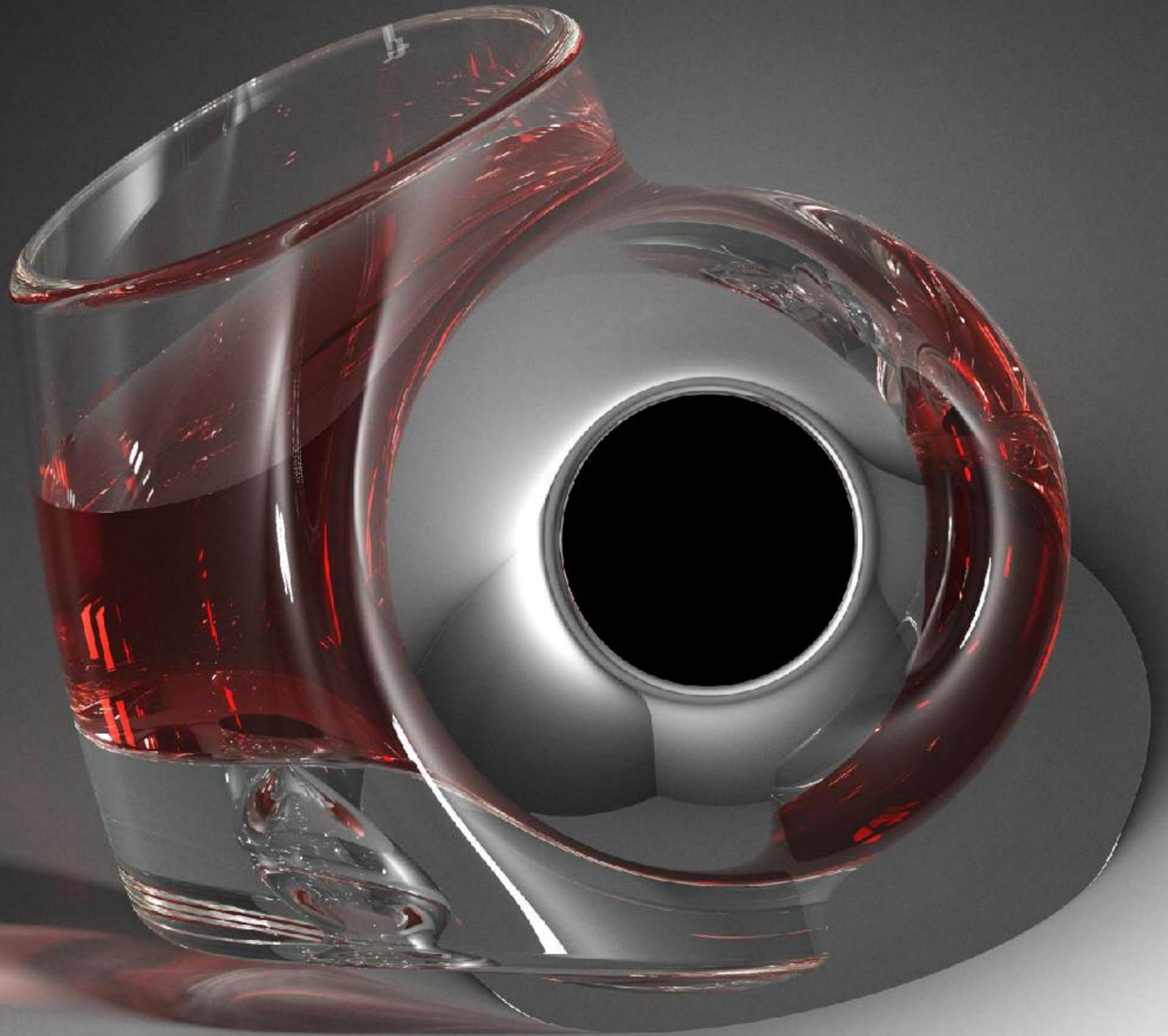






III

Path Tracing

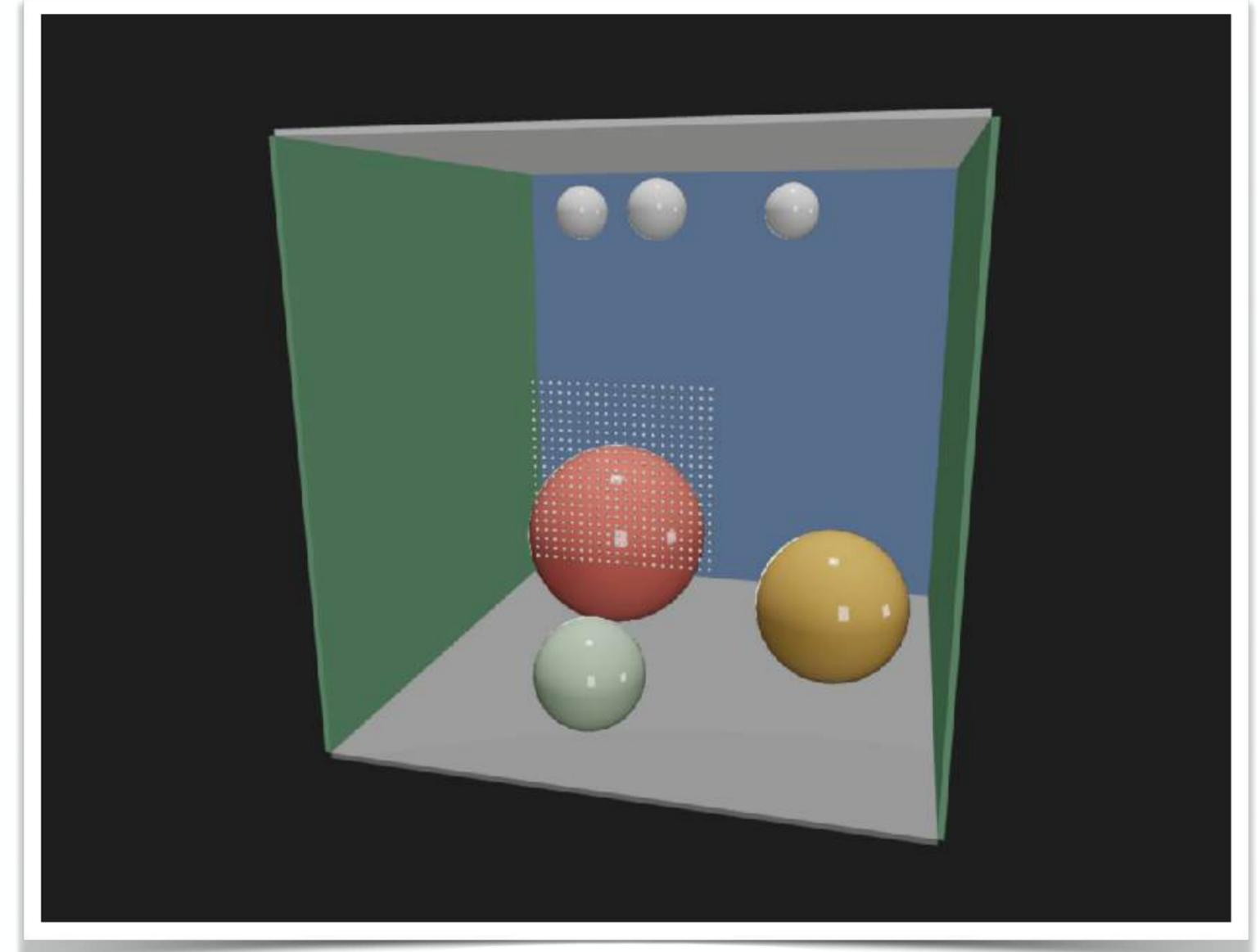
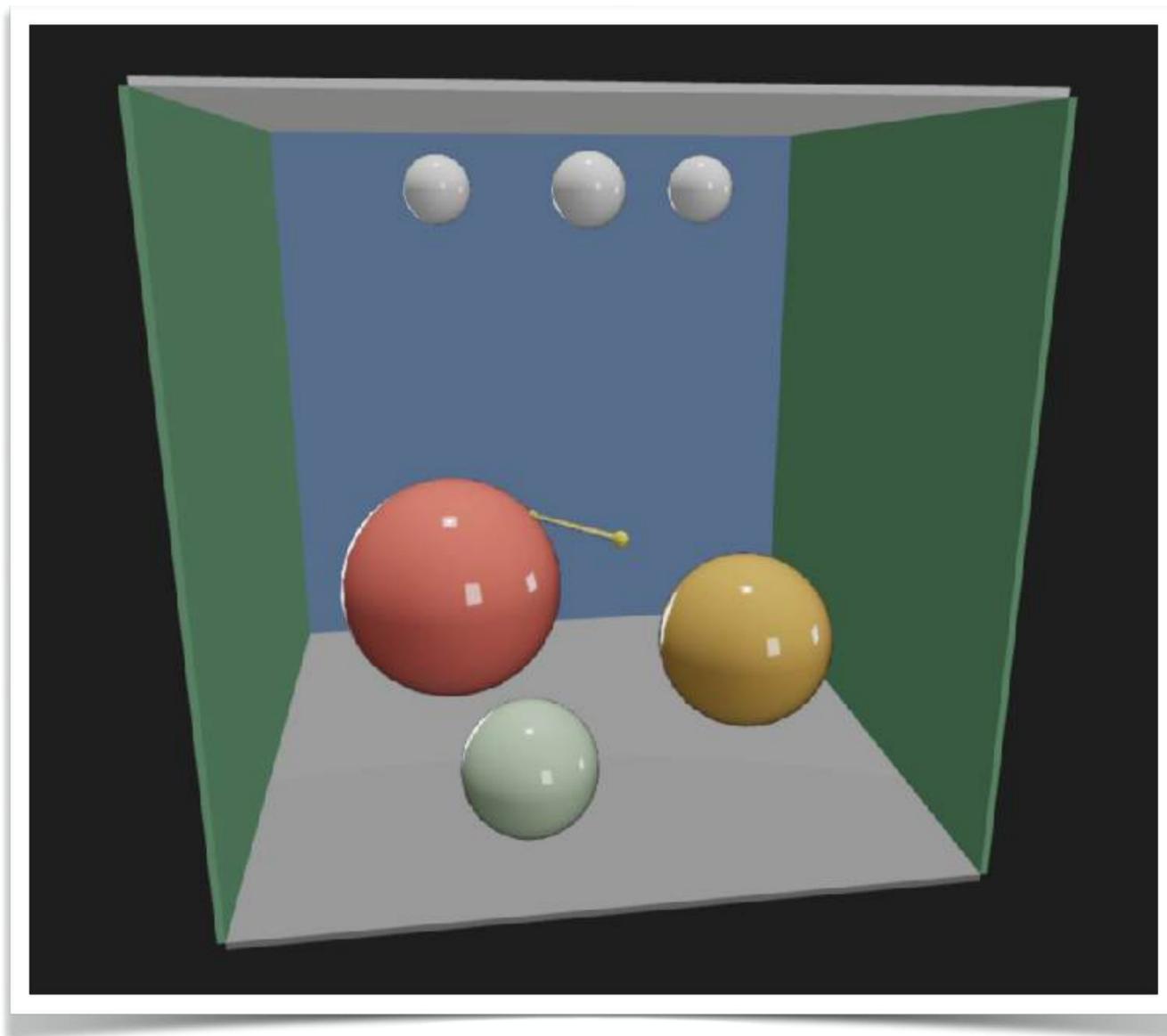


The Path Tracing Algorithm

Making a realistic image by

Following Geodesics

Interacting with Objects



Following Geodesics

$$U = 1 + \sum_i \frac{m_i}{r_i}$$

$$g = U^4(dx^2 + dy^2 + dz^2)$$

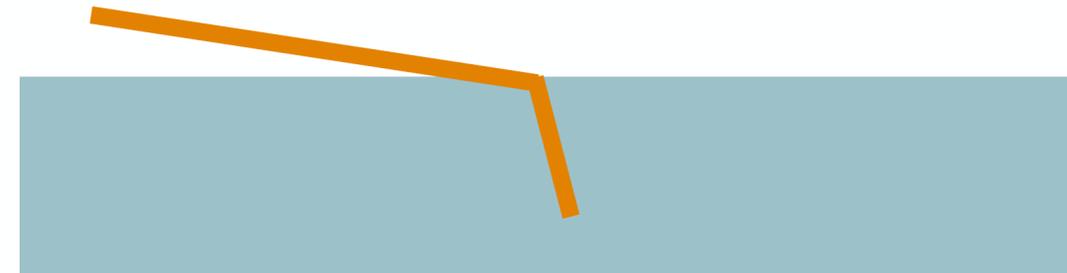
The Geodesic Equation

$$U\ddot{\gamma} = 2\|\dot{\gamma}\|^2\nabla U - 4(\dot{\gamma} \cdot \nabla U)\dot{\gamma}$$

Interacting with Objects



$$v \mapsto v - 2\frac{g(v, n)}{g(n, n)}n$$



$$rv - \left(r\frac{g(v, n)}{g(n, n)} - \sqrt{1 - r^2 \left(1 - \frac{g(v, n)^2}{g(n, n)} \right)} \right) n$$

Following Geodesics

$$U = 1 + \sum_i \frac{m_i}{r_i}$$

$$g = U^4(dx^2 + dy^2 + dz^2)$$

The Geodesic Equation

$$U\ddot{\gamma} = 2\|\dot{\gamma}\|^2\nabla U - 4(\dot{\gamma} \cdot \nabla U)\dot{\gamma}$$

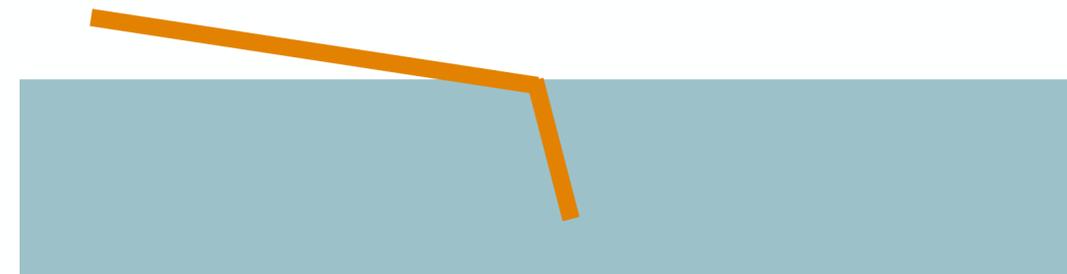
Interacting with Objects



$$v \mapsto v - 2 \frac{g(v, n)}{g(n, n)} n$$

Conformal to the Euclidean Metric

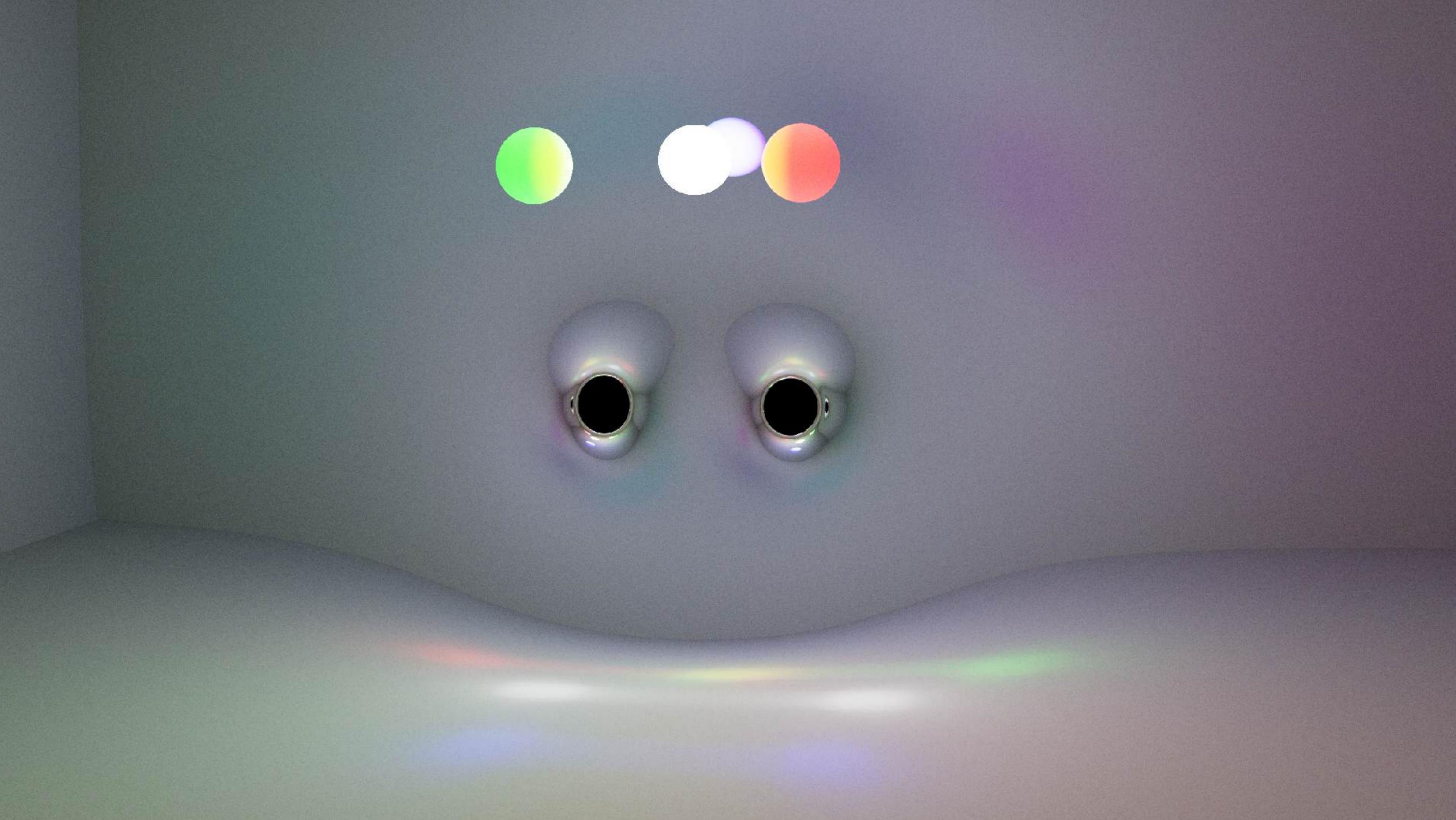
Can use Euclidean Dot Product!



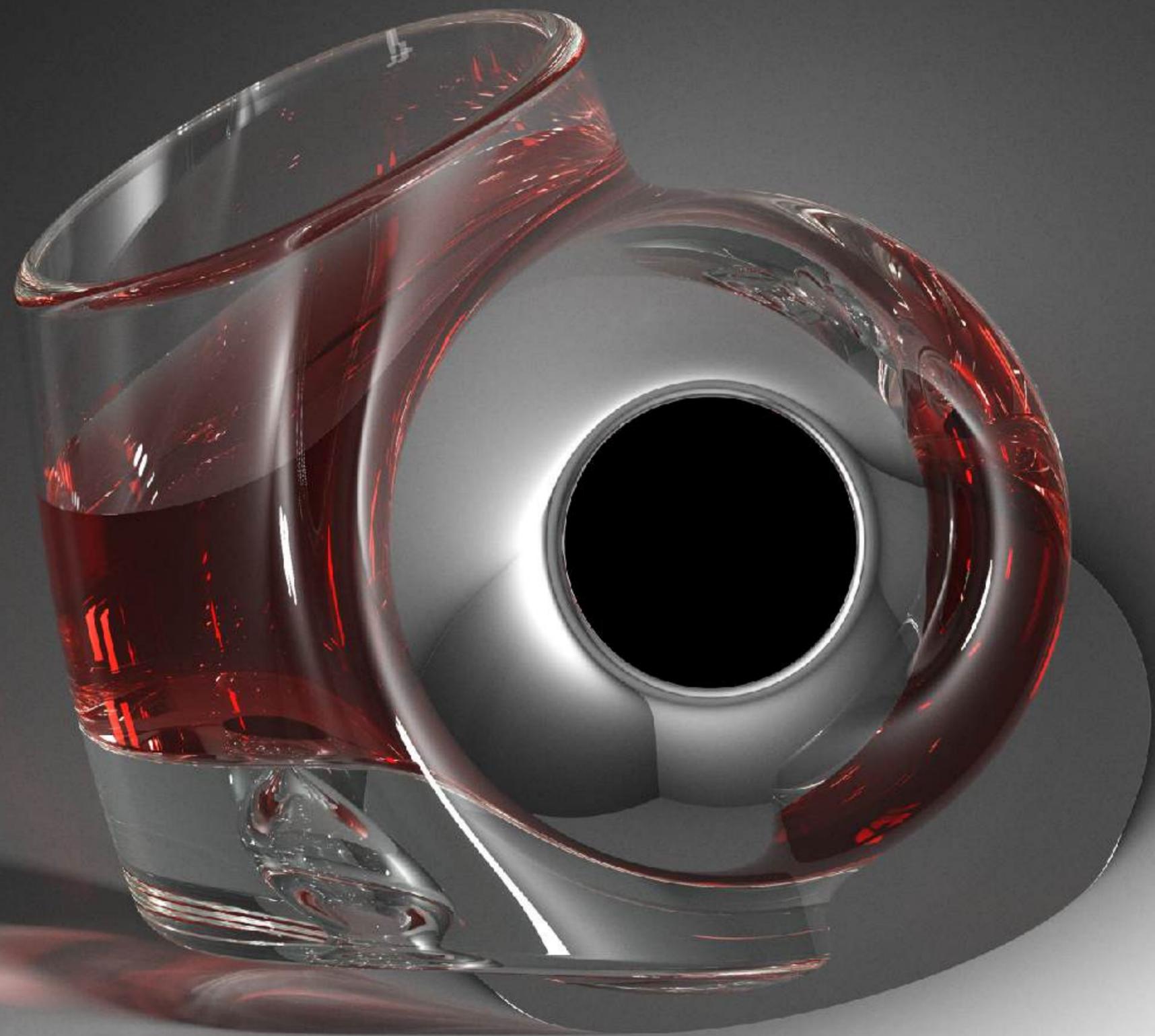
$$rv \mapsto \left(r \frac{g(v, n)}{g(n, n)} - \sqrt{1 - r^2 \left(1 - \frac{g(v, n)^2}{g(n, n)} \right)} \right) n$$





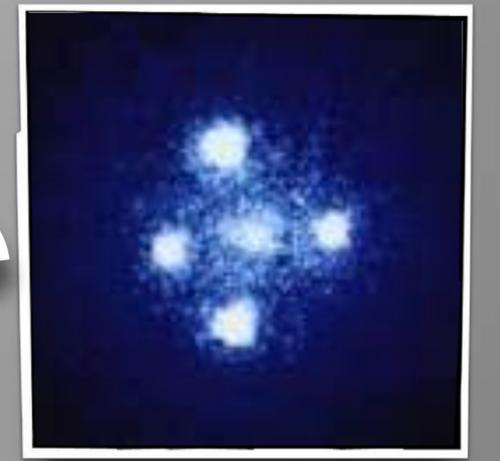








Einstein Cross



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