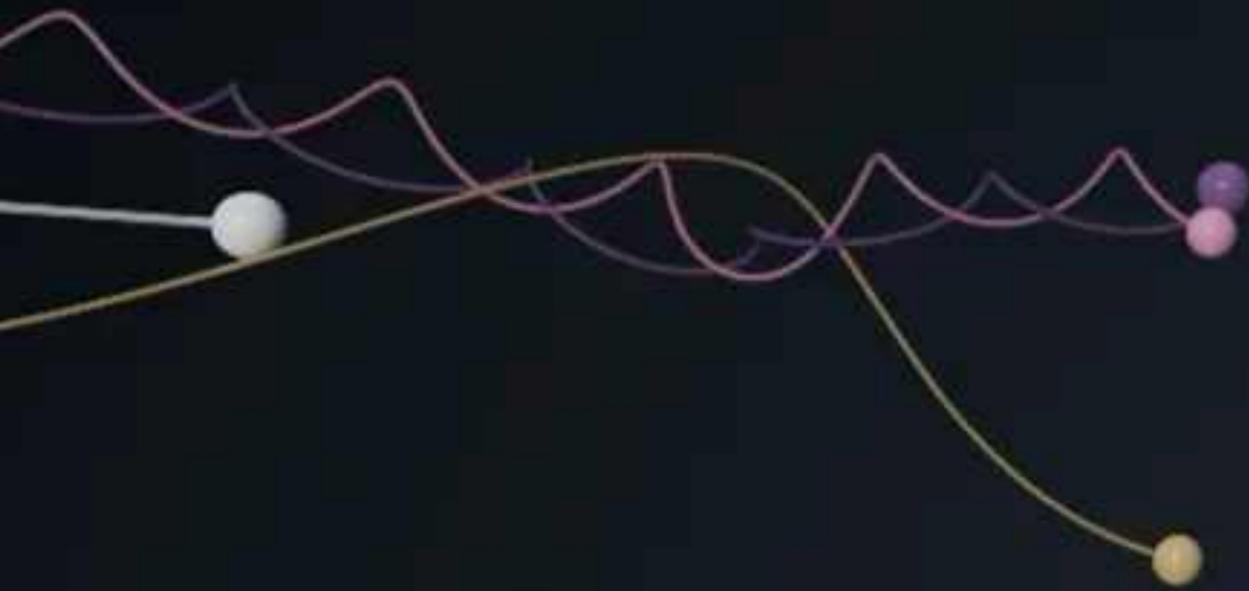


Steve Trettel

CHAOS



SPACE INVADERS

TECHNOLOGY ON EARTH



WHAT HAPPENED?!

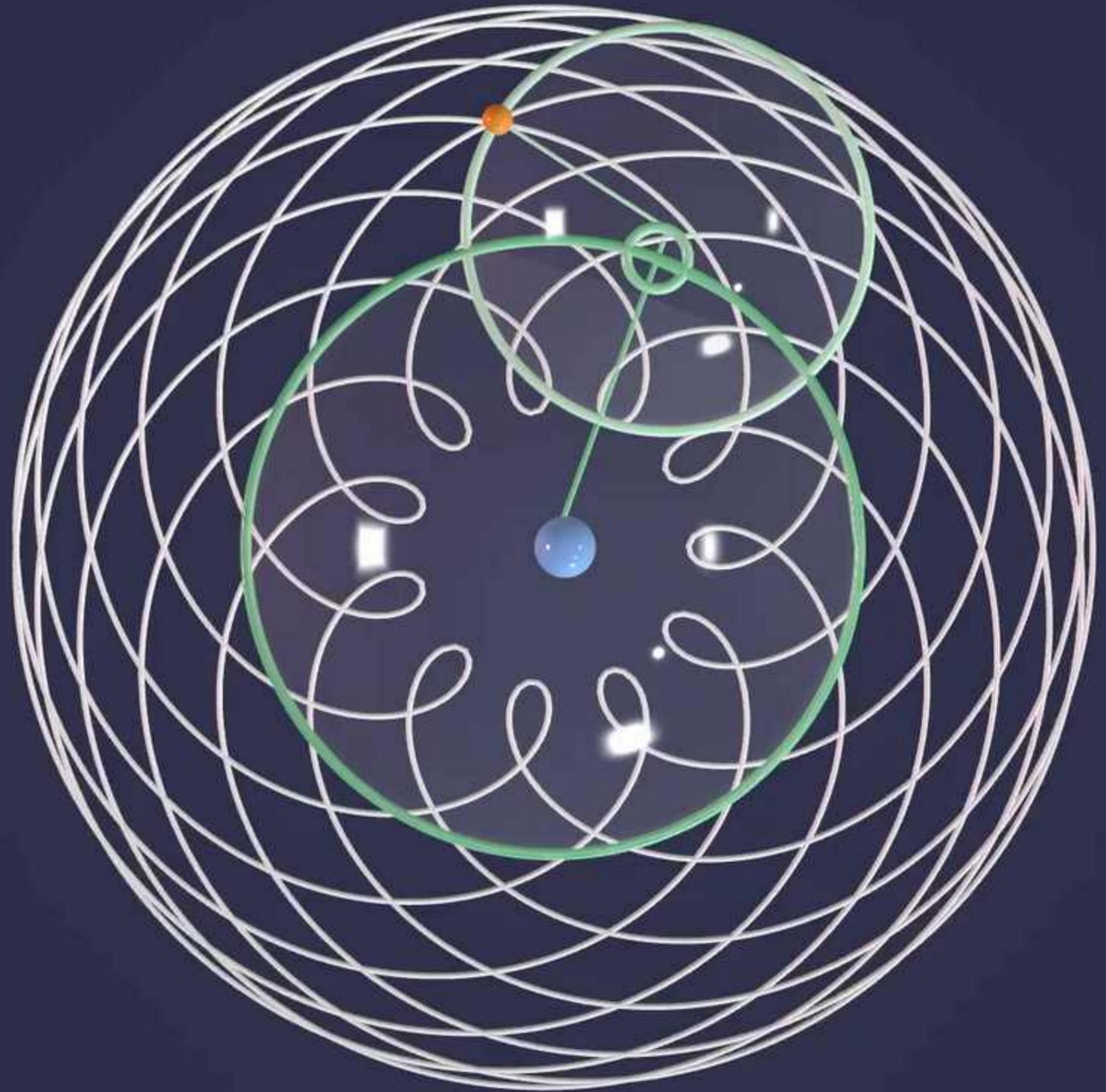
We discovered
that physics is
predictable!

Nature follows
mathematical laws.



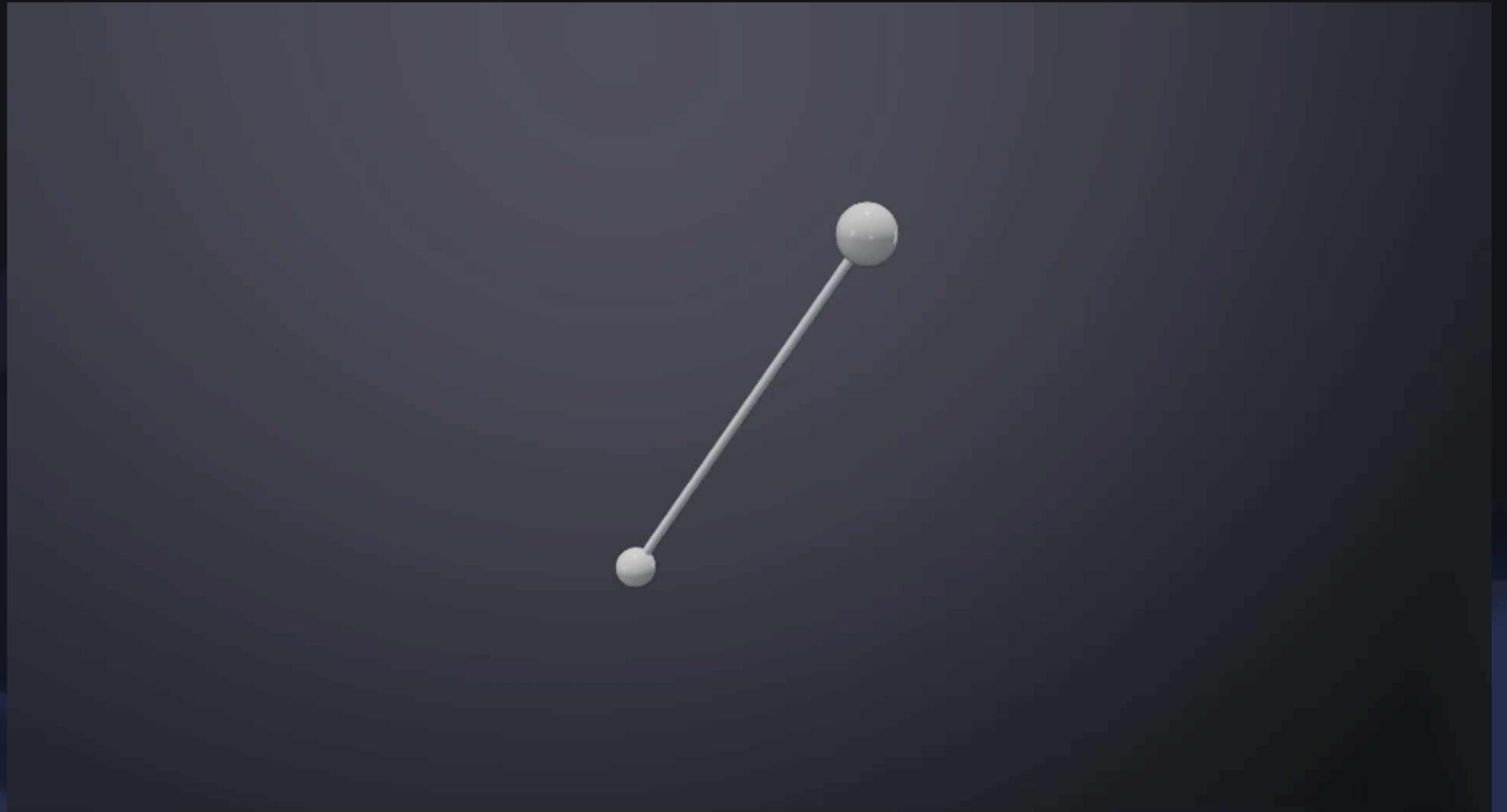
Πτολεμαῖος

Regularities in the
solar system.

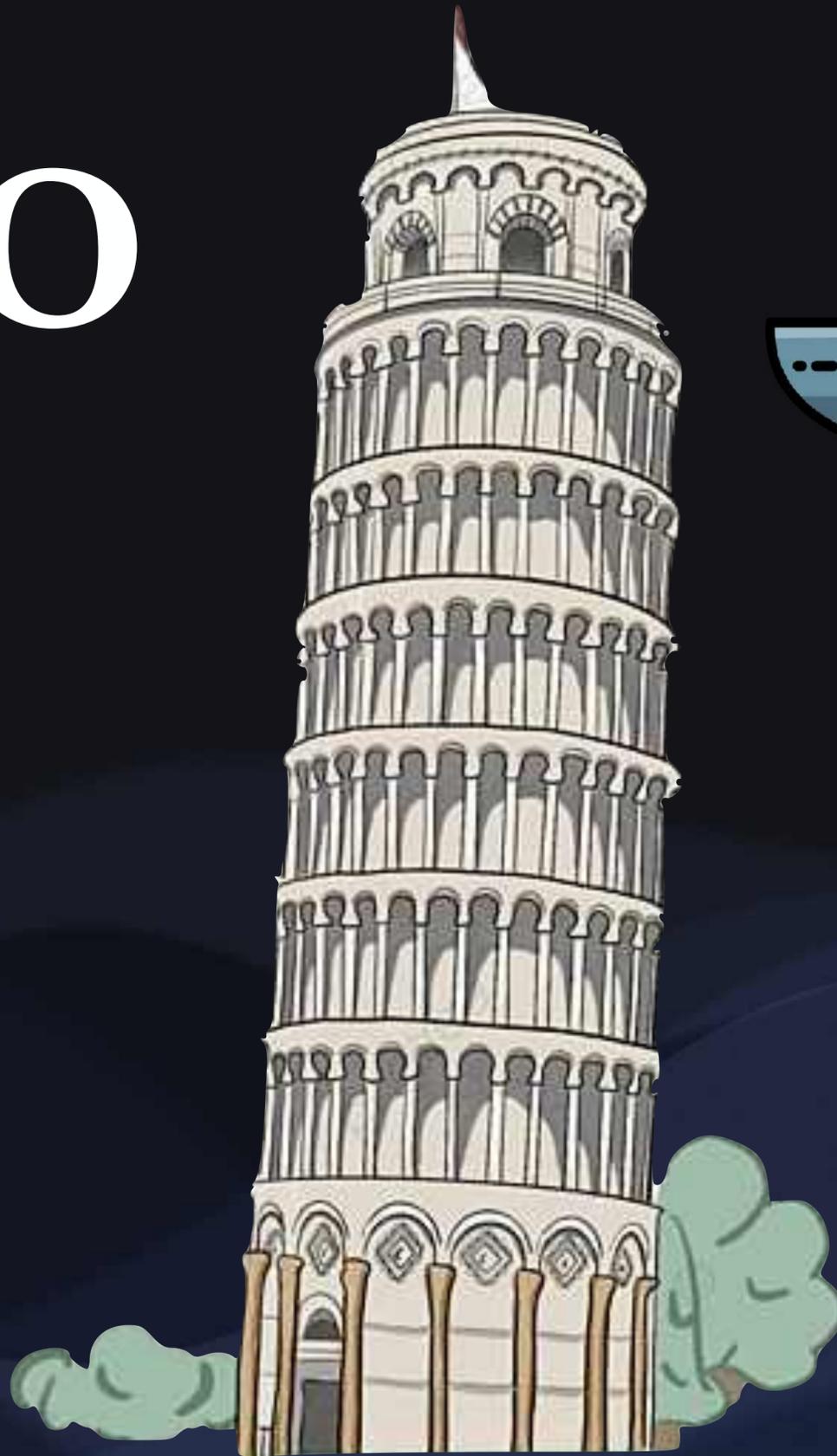


GALILEO

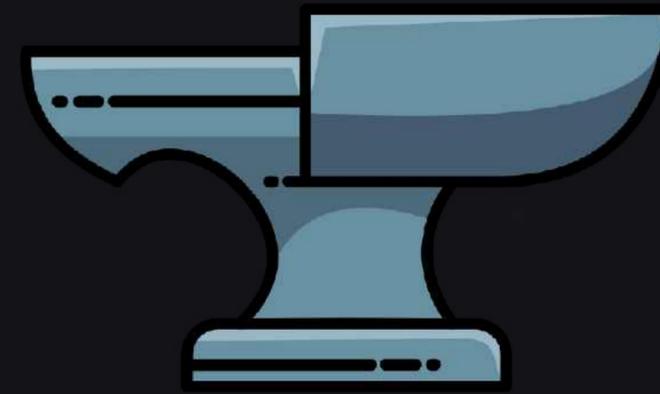
Universal period of the pendulum.



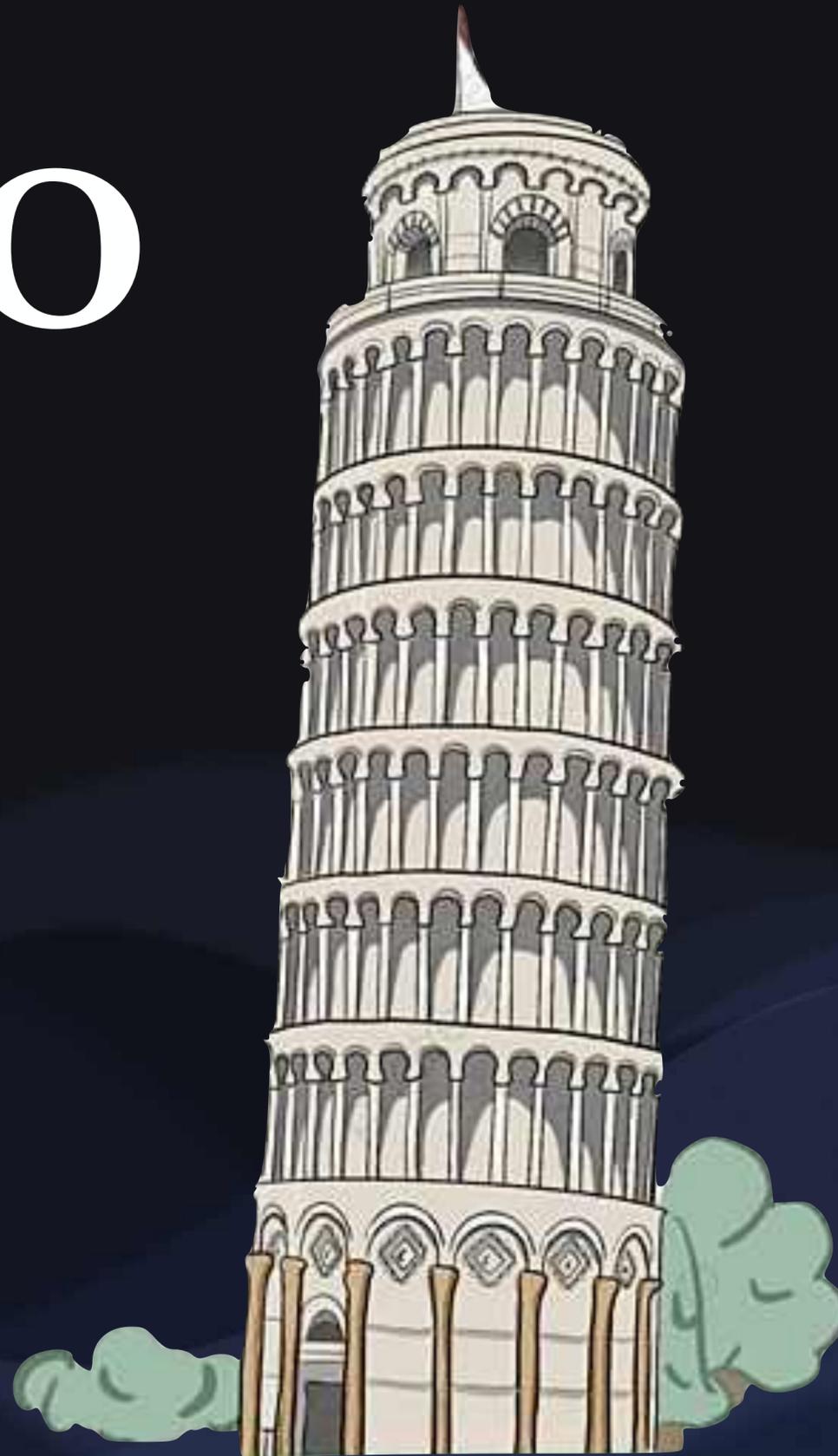
GALILEO



Universal
Rate of Fall



GALILEO



1



3

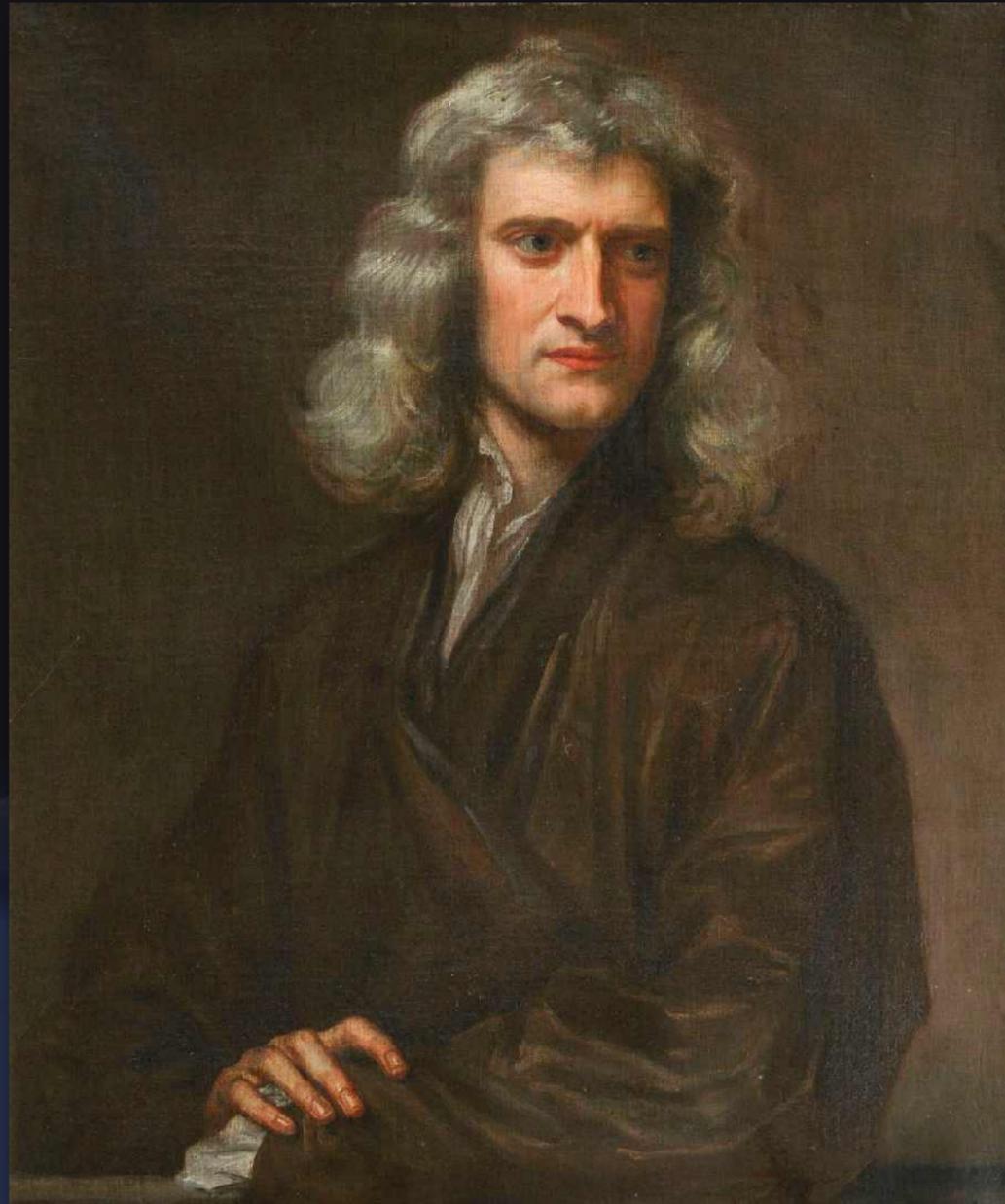


5



Law of
Odd
Numbers

NEWTON



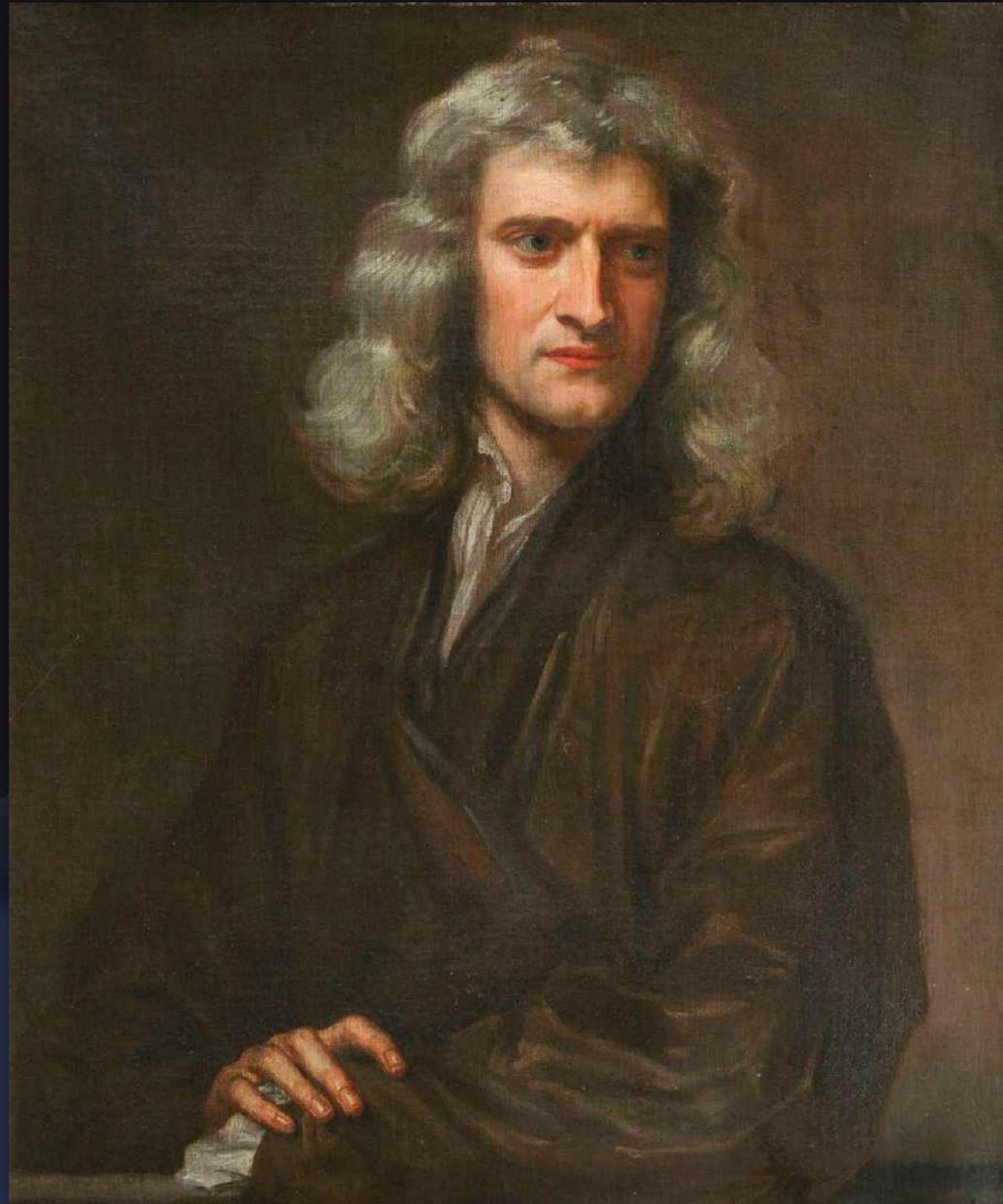
Structure of Physical Law

*What is happening right now
determines what happens next.*

Force and Acceleration

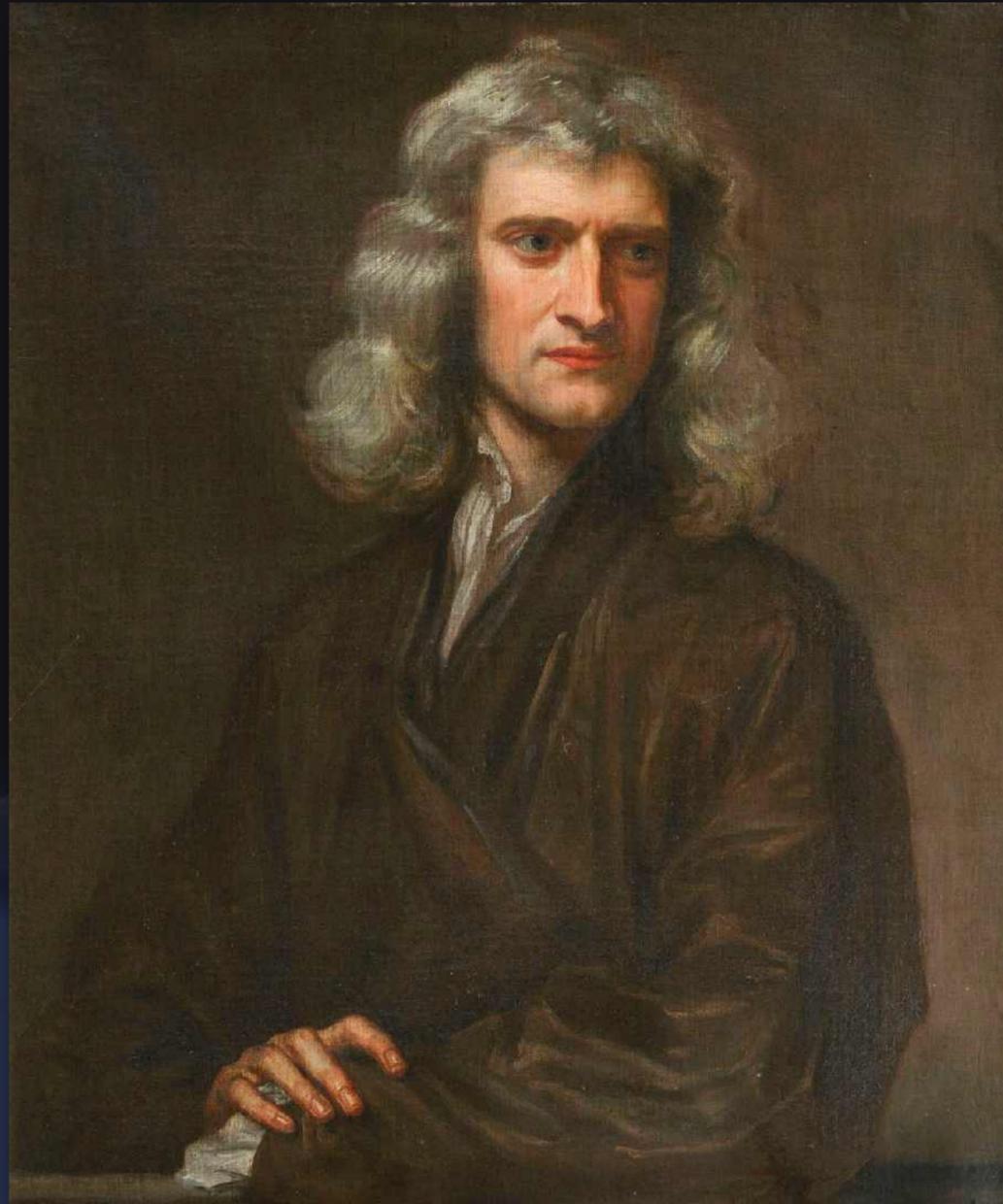
$$\mathbf{F} = ma$$

The Long Shadow of Newton



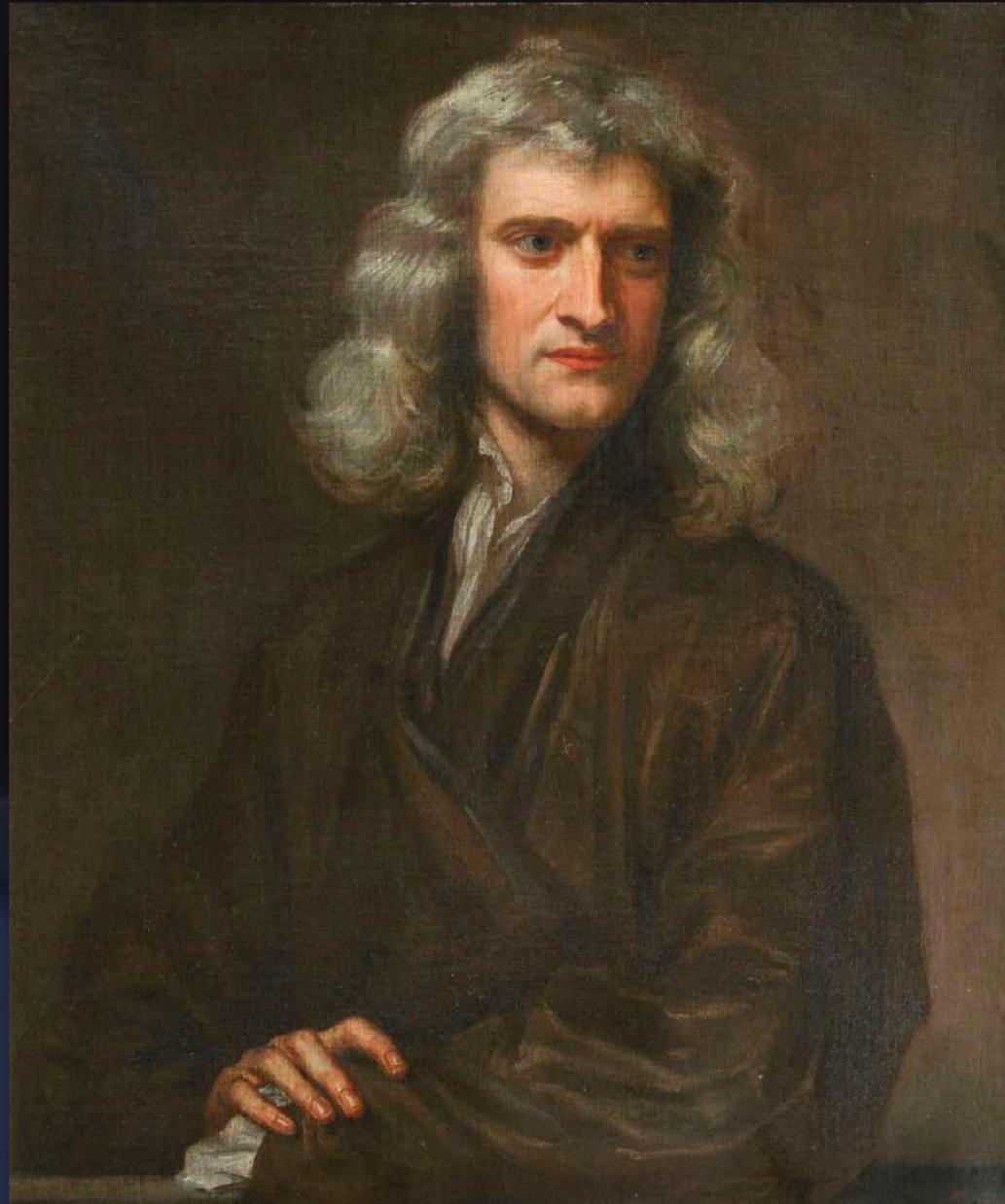
*What is happening right now
determines what happens next.*

The Long Shadow of Newton



*What is happening right now
determines
the rate that things change*

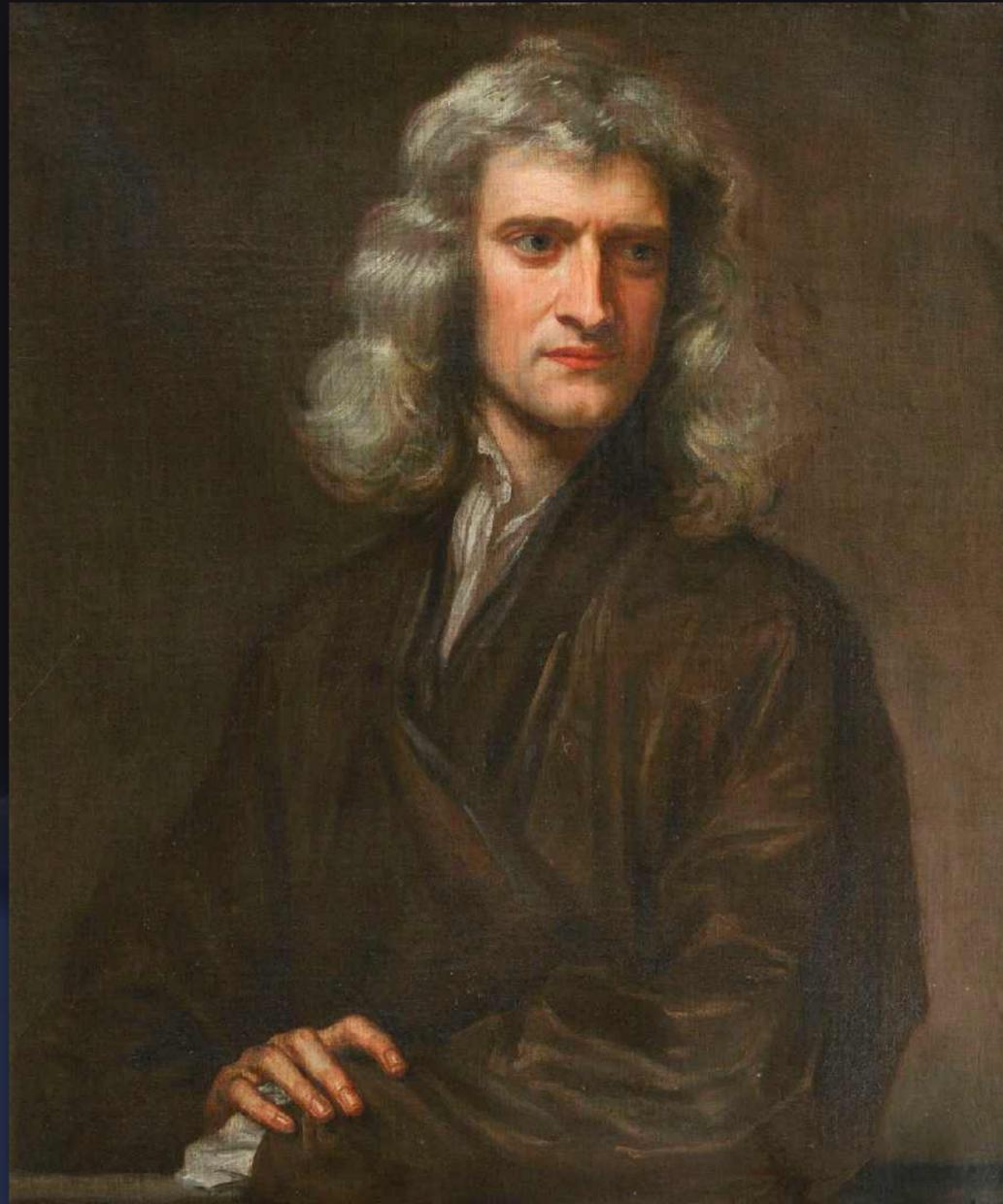
The Long Shadow of Newton



*The current state of a system
determines*

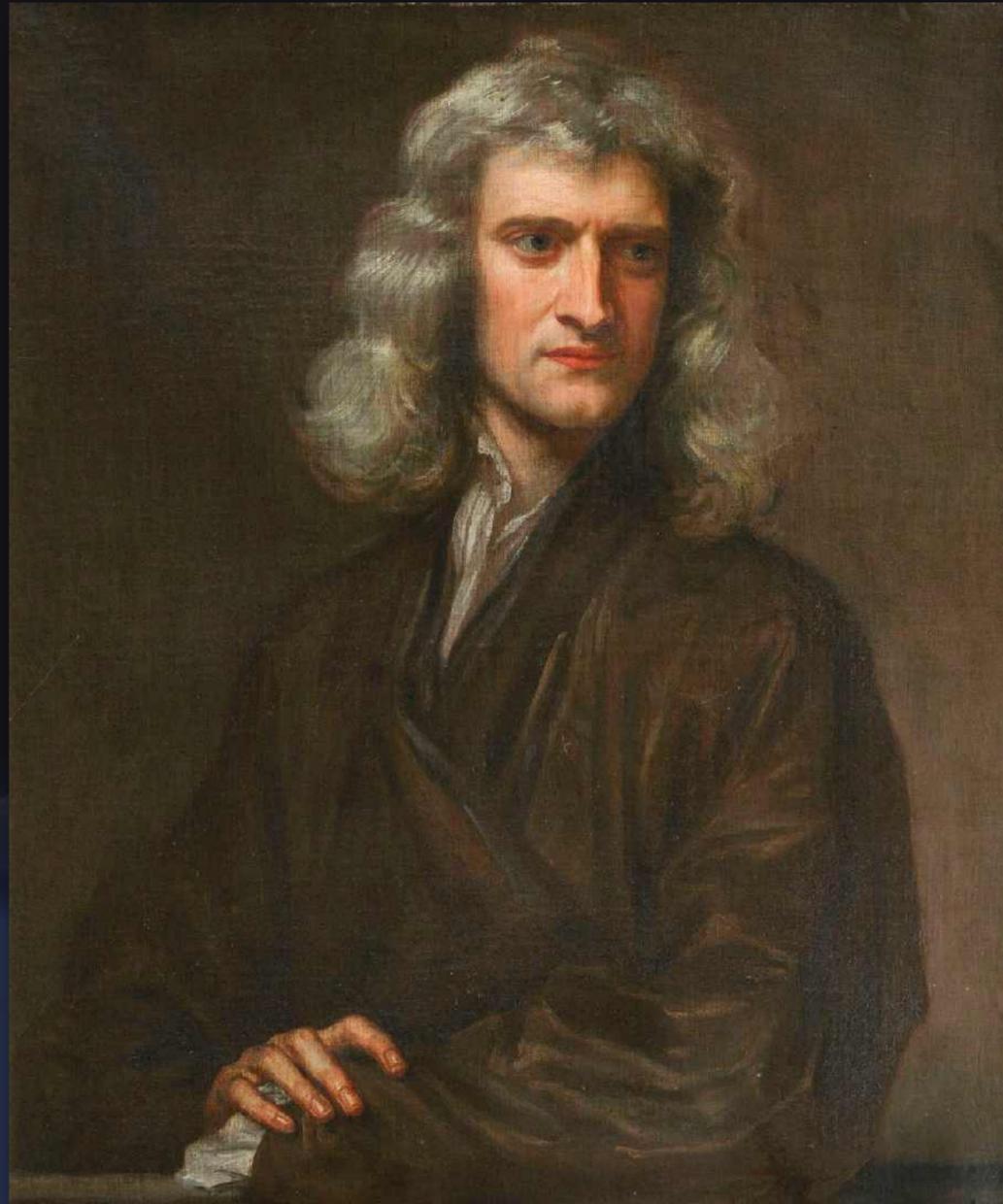
the rate that things change

The Long Shadow of Newton



*The current state of a system
is related to **its derivatives**
by an equation*

The Long Shadow of Newton



*Physical systems
are governed by
differential equations*

The Long Shadow of Newton

*Physical systems
are governed by
differential equations*

Solve for y :

$$yy' = t^2 + C$$

$$y y' = 1 + t + C$$

$$y' = Ce^t$$

$$y y' = \ln(t) + C$$

The Long Shadow of Newton

Physics is **predictive!**

Given a physical system

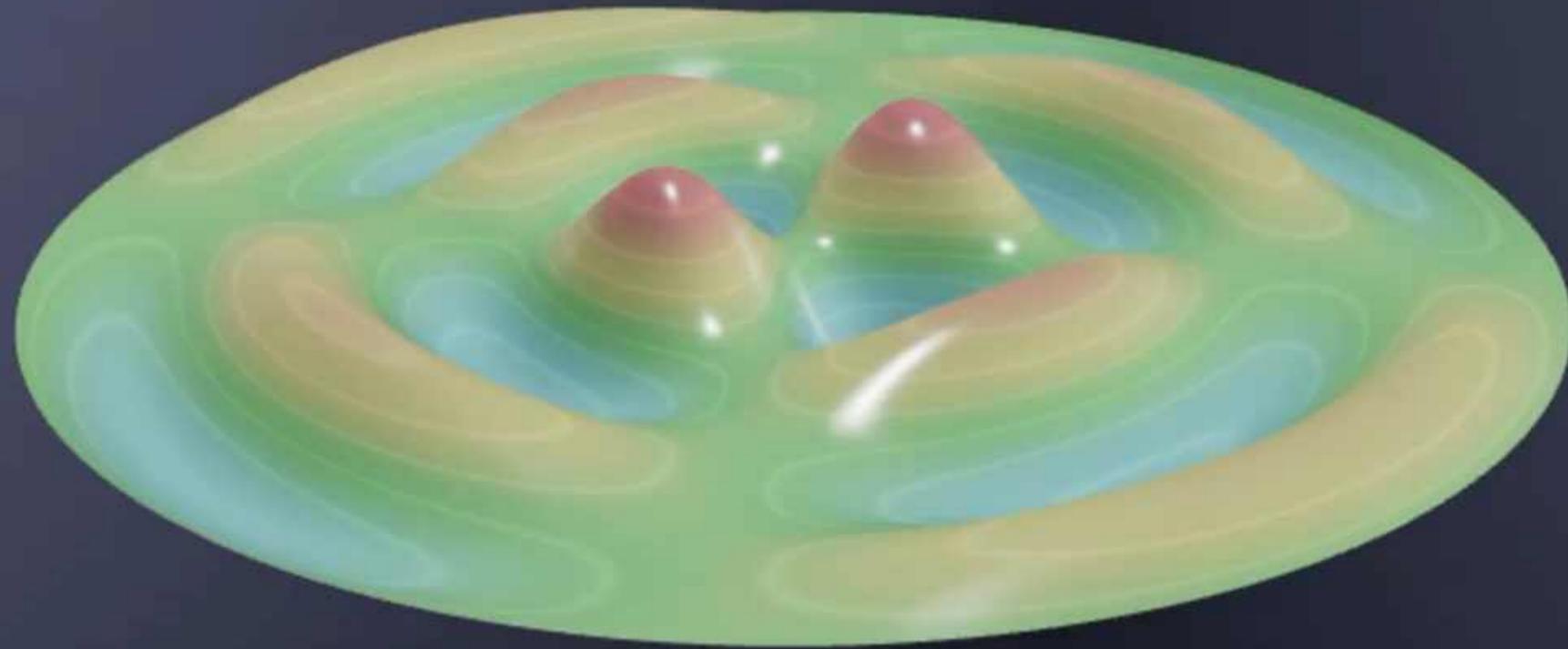
Physics produces a differential equation
Calculus accurately computes the future.

Celestial Mechanics



$$\mathbf{r}'' = -\frac{GM}{|r|^3}\mathbf{r}$$

Material Science



$$\partial_t^2 W = (\partial_x^2 + \partial_y^2) W$$

Temperature

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}$$



Statistical Mechanics



$$T = \frac{\partial E}{\partial S}$$

Fluid Mechanics

$$\nabla \cdot \mathbf{v} = 0$$

$$\rho \frac{D\mathbf{v}}{Dt} = -\nabla p + \rho \mathbf{g} + \mu \nabla^2 \mathbf{v}$$

Electromagnetism

$$(c^2 \nabla^2 - \partial_t^2) \mathbf{B} = 0$$

$$(c^2 \nabla^2 - \partial_t^2) \mathbf{E} = 0$$

General Relativity

$$\text{Ric} = 8\pi G \left(\mathbf{T} - \frac{1}{2} T \mathbf{g} \right)$$

Quantum Mechanics

$$-i \frac{\partial \Psi}{\partial t} = H \Psi$$

PART I

Mathematical
Physics

Physics:

Understand the natural world,
through experiments and
mathematical models.

Mathematical Physics:

Understand the mathematical models
arising in physics, using mathematics
(theorems, conjectures, proofs).

Galileo & Parabolas



1

$$1 + 3 = 4^2$$



3

$$1 + 3 + 5 = 9^2$$



5

$$1 + 3 + 5 + 7 = 16^2$$



$$1 + 3 + 5 + 7 + 9 = 25^2$$

$$1 + 3 + 5 + 7 + 9 + 11 = 36^2$$

Fall Time

$$\propto (\textit{time})^2$$

Galileo & Parabolas



Mathematical Physics:

*Prove that in **Newtonian mechanics**, a point particle follows a parabola under a constant downwards acceleration.*

Galileo & Parabolas

$$\int y'' dt = \int -g dt + C$$



Galileo & Parabolas

$$\int y' dt = \frac{g}{2} t^2 + vt + k$$

Galileo & Parabolas

$$y = \frac{g}{2}t^2 + vt + h$$

Parabola!

Tycho Brahe & Kepler

Newton & Orbits

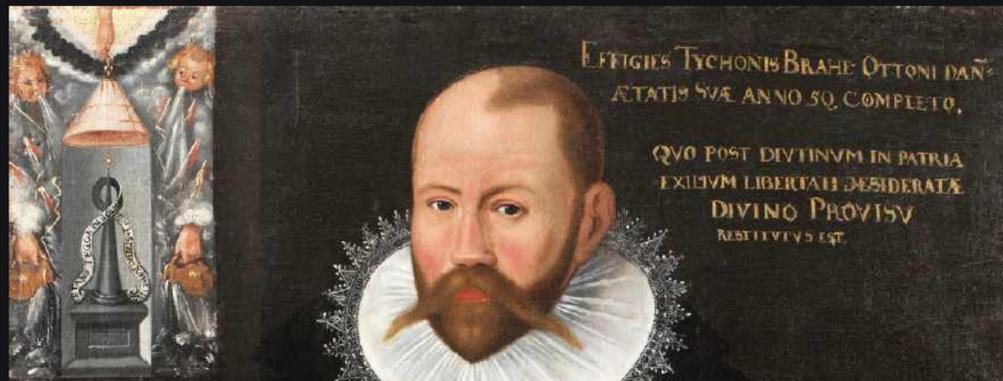


nis Lune.

Addit. Dimitt.		3		4		Signa								
Centi et Argumenti veri	Argumenti	ADmitta proportionalia	Diferentia Diametri.	Equatio Argumenti.	Corridos unum minuto.	Argumenti Centi.	ADmitta proportionalia.	Diferentia Diametri.	Equatio Argumenti.	Corridos unum minuto.	Centi et Argumenti veri			
g m	g m	m	g m	g m fa fa f.	g m m	g m	g m m	g m	g m fa fa f.	g m m	g m			
0 0	12 0	26	2 16	4 54 54	0 24	13 4 41	2 31	4 27 0	2 18	9 22 55	1 35	2 19 35	4 42	10 0
0 10	12 1	26	2 16	4 54 58		13 4 41	2 31	4 26 17		9 20 55	1 35	2 18 48		29 56
0 20	12 2	26	2 16	4 55 2		13 4 41	2 31	4 26 14	2 24	9 18 55	1 34	2 18 1		29 46
0 30	12 3	26	2 16	4 55 6		13 4 41	2 31	4 25 50		9 15 55	1 34	2 17 14		29 36
0 40	12 4	26	2 16	4 55 10		13 3 43	2 30	4 25 26		9 13 56	1 33	2 16 27		29 26
0 50	12 4	27	2 17	4 55 14		13 3 43	2 30	4 25 2		9 11 56	1 33	2 15 40		29 16
1 0	12 5	27	2 17	4 55 18	0 18	13 3 43	2 30	4 24 38		9 8 56	1 32	2 14 52	4 48	29 0
1 10	12 6	27	2 17	4 55 22		13 3 43	2 30	4 24 14		9 6 56	1 32	2 14 5		28 56
1 20	12 7	27	2 17	4 55 25		13 2 43	2 30	4 23 50		9 3 56	1 31	2 13 17		28 46
1 30	12 8	27	2 17	4 55 28		13 2 43	2 30	4 23 26	2 30	9 1 56	1 31	2 12 30		28 36
1 40	12 8	27	2 17	4 55 31		13 2 44	2 29	4 23 1		8 58 56	1 30	2 11 42		28 26
1 50	12 9	27	2 18	4 55 34		13 1 44	2 29	4 22 36		8 56 56	1 30	2 10 54		28 16
2 0	12 10	27	2 18	4 55 37	0 12	13 1 44	2 29	4 22 11		8 51 56	1 29	2 10 6		28 0
2 10	12 11	27	2 18	4 55 39		13 1 44	2 29	4 21 46		8 51 56	1 29	2 29 18		27 56

Tycho Brahe & Kepler

Newton & Orbits



nis Lune.

Addit.	3	4	5	Signa
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Handwritten manuscript text in Latin, likely Kepler's work on planetary motion.

1 50	12 9	27	2 38	4 55 34	13 1 44	2 2
2 0	12 10	27	2 38	4 55 37	0 12 13	1 44
2 10	12 11	27	2 38	4 55 39	13 1 44	2 2

COPERNICI

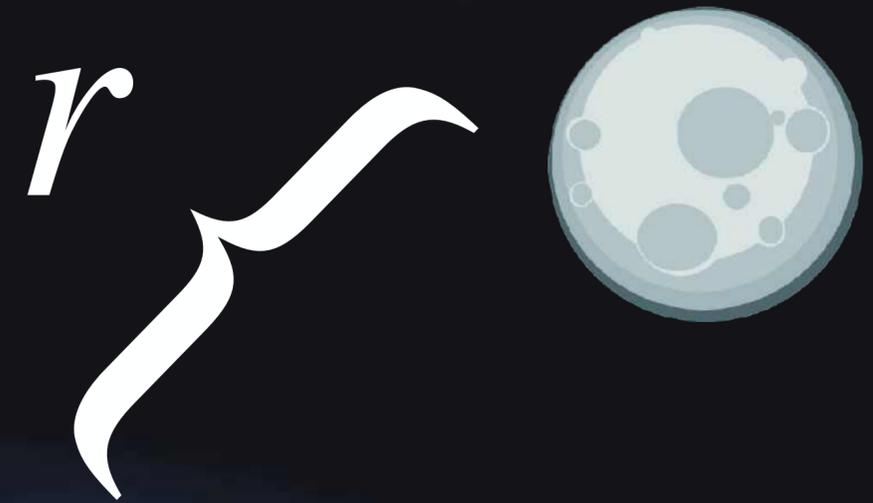
PTOLEMEI

PRIMUM hęc in forma COPERNICANA ut simpli- ciori ad sensum propone- mus.

Sed a punctum aqua- litatis circuli terra, qui- punctus est circuli hęc ex a descriptus: Et si Sol in puncto hęc a hęc a puncto

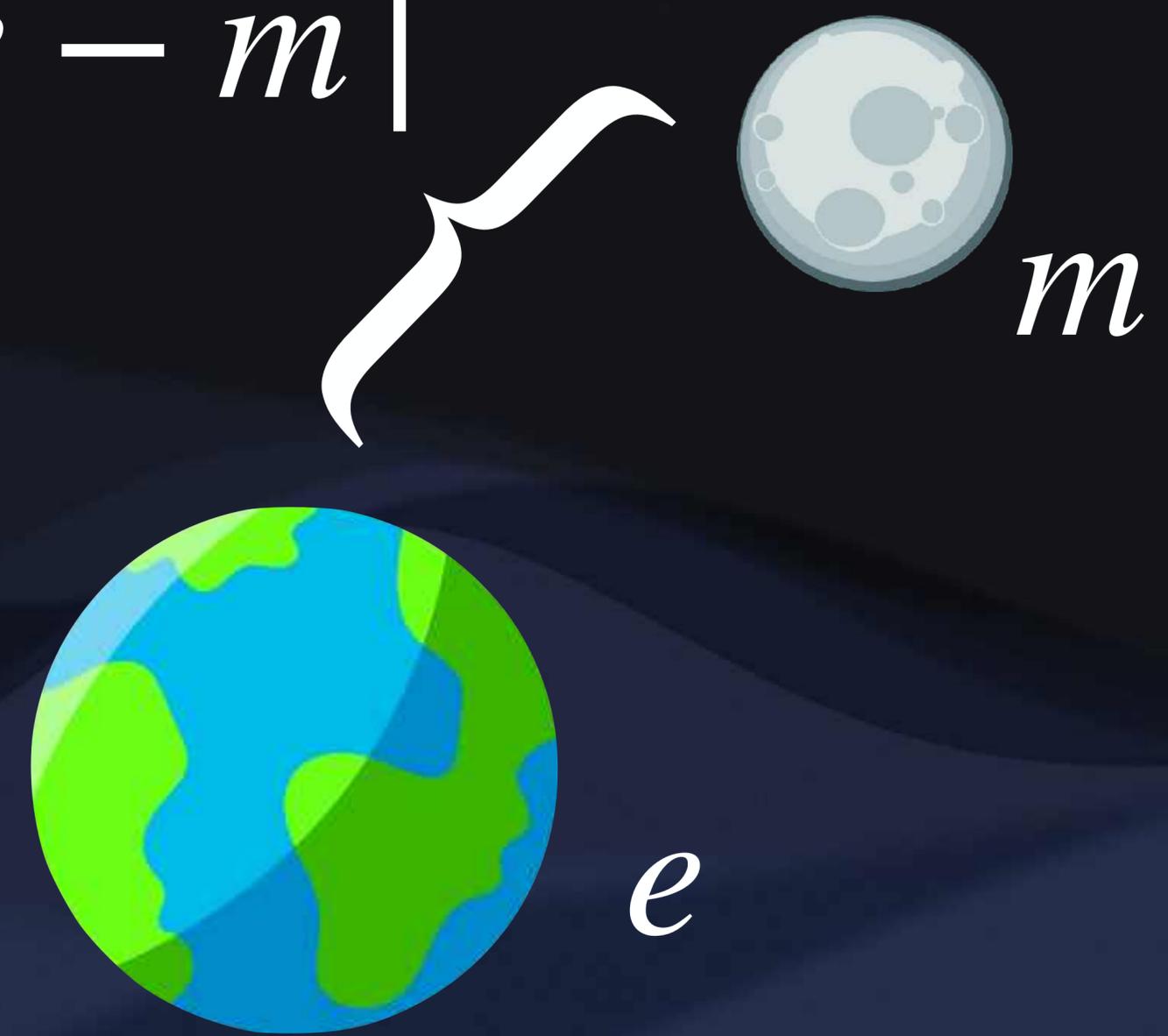
Hooke & Newton

Newton & Orbits



$$|F| \propto \frac{1}{r^2}$$

Newton & Orbits

$$|\mathbf{F}_e| = \frac{GM_e M_m}{|e - m|^2}$$


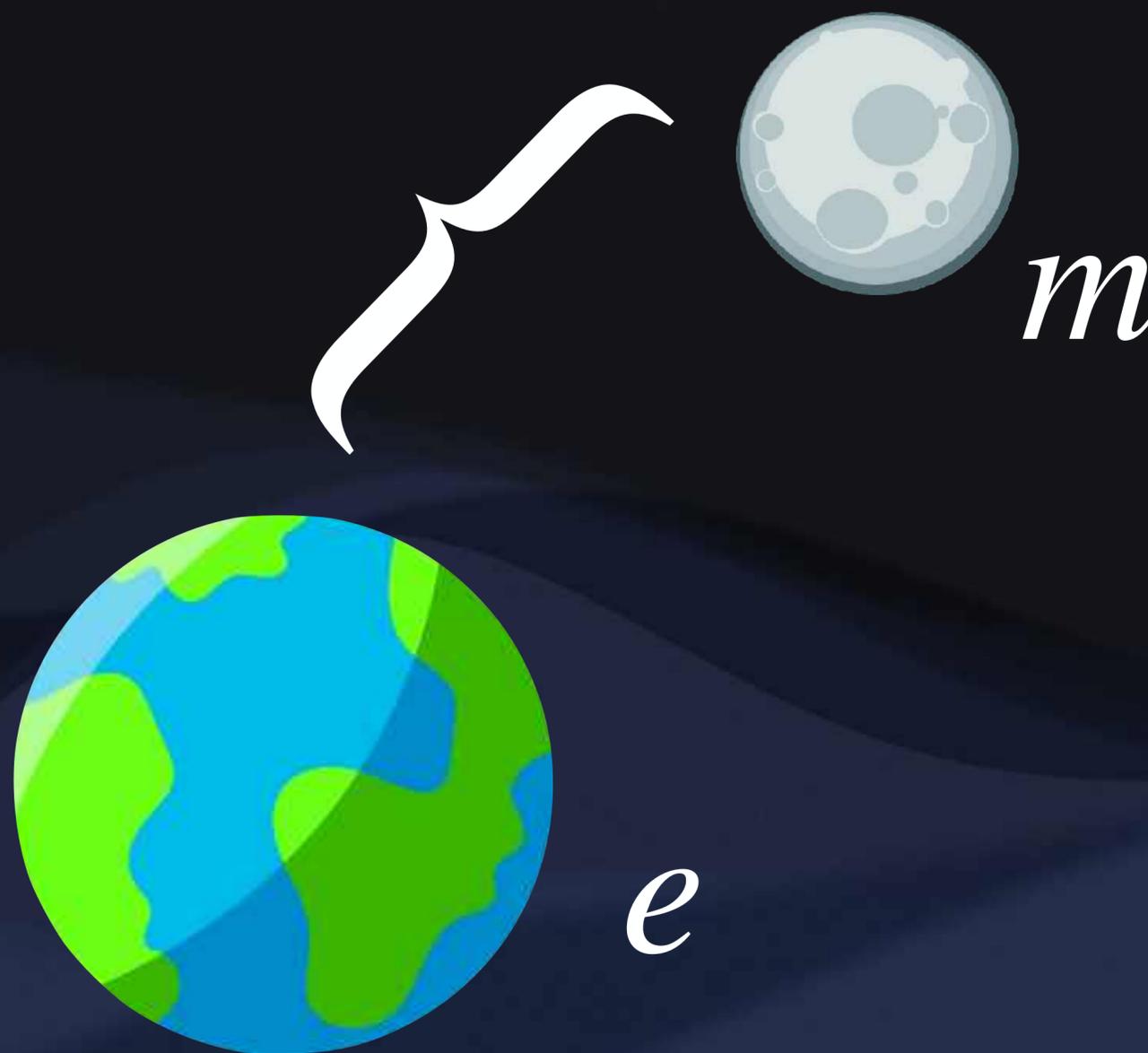
The diagram illustrates the gravitational force between Earth and the Moon. Earth is shown as a blue and green sphere labeled e at the bottom. The Moon is shown as a grey sphere with craters labeled m at the top right. A white curly brace indicates the distance between the centers of Earth and the Moon, labeled $|e - m|$ above it.

Newton & Orbits

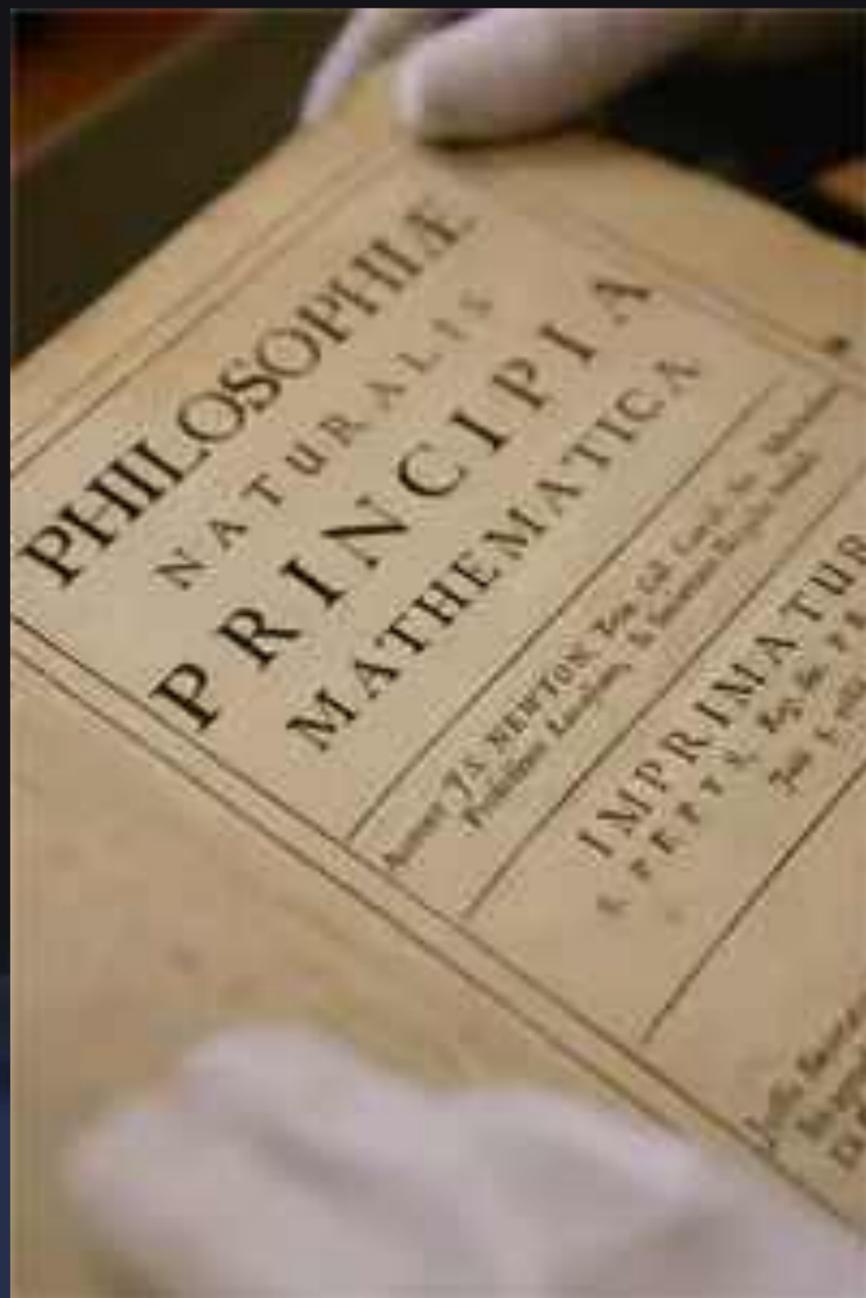
Mathematical Physics:
Prove the solution to *these equations* (Newton's law of gravity) are *ellipses*

$$e'' = \frac{-GM_m}{|e - m|^2} \widehat{e - m}$$

$$m'' = \frac{-GM_e}{|m - e|^2} \widehat{m - e}$$



Newton & Orbits



PRINCIPIA MATHEMATICA. 345
LIBER SECUNDUS

S E C T I O IX.
De Motu Circulari Fluidorum.

HYPOTHESIS.

Resistentiam, quæ oritur ex defectu lubricitatis partium Fluidi, cæteris paribus, proportionalem esse velocitati, qua partes Fluidi separantur ab invicem.

PROPOSITIO LI. THEOREMA XXXIX.

Si Cylindrus solidus infinite longus in Fluido uniformi & infinito circa axem positione datum uniformi cum motu revolvatur, & ab hujus impulsu solo agatur Fluidum in orbem, perseveret autem Fluidi pars unaquæque uniformiter in motu suo; dico quod tempora periodica partium Fluidi sunt ut ipsarum distantie ab axe Cylindri.

Sit *AFL* Cylindrus uniformiter circa axem *S* in orbem actus, & circulis concentricis *BGM, CHN, DIO, EKP*, &c. distinguatur Fluidum in Orbes Cylindricos innumeros concentricos solidos ejusdem crassitudinis. Et quoniam homogeneum est Fluidum, impressiones contiguorum Orbium in se mutuo factæ, erunt (per Hypothesin) ut eorum translationes ab invicem & superficies contiguæ in quibus impressiones fiunt. Si impressio in Orbem aliquem major est vel minor ex parte concava quam ex parte

X x

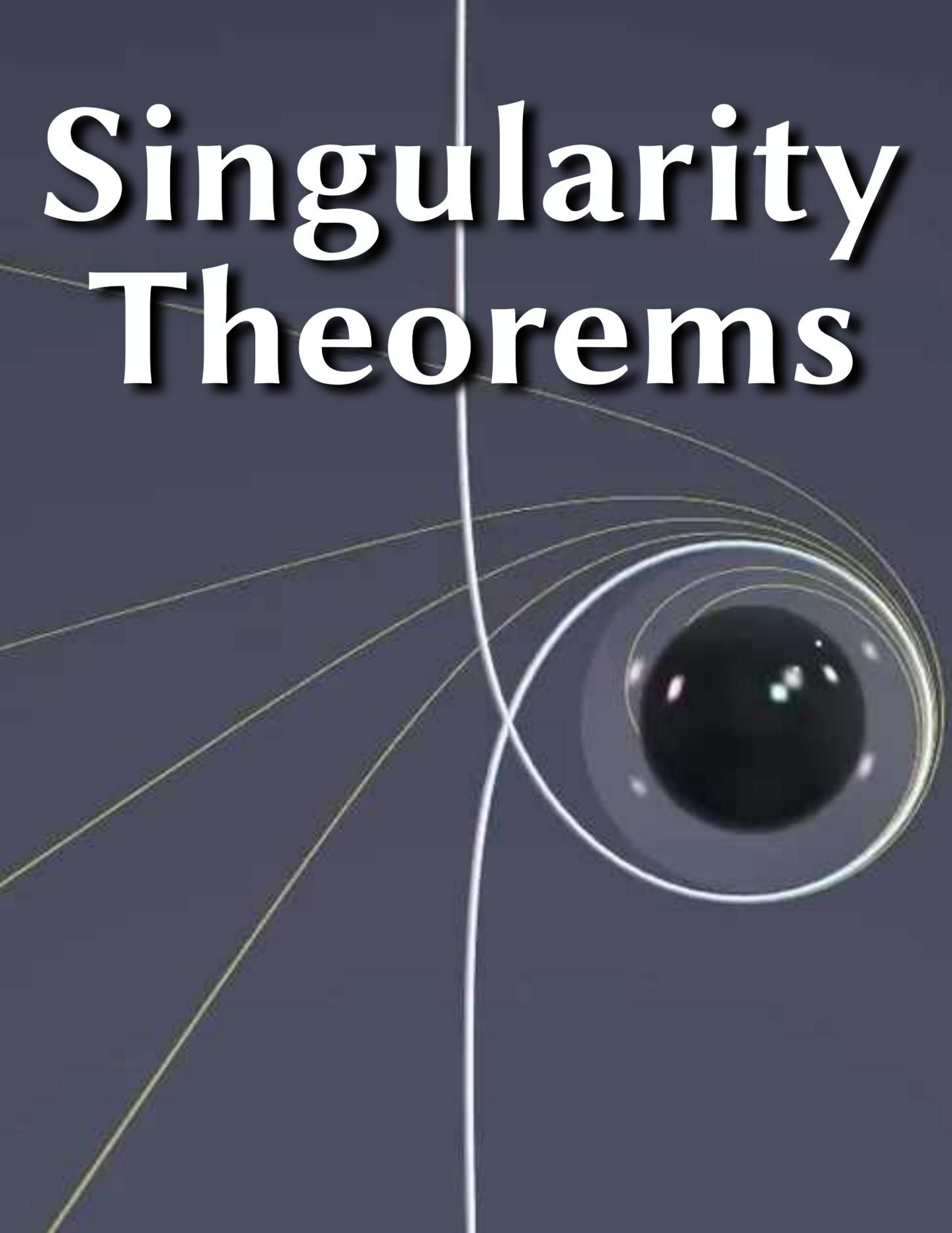
Theorem:
They are! In fact, all two-body systems follow *conic sections*.



Newton & Orbits

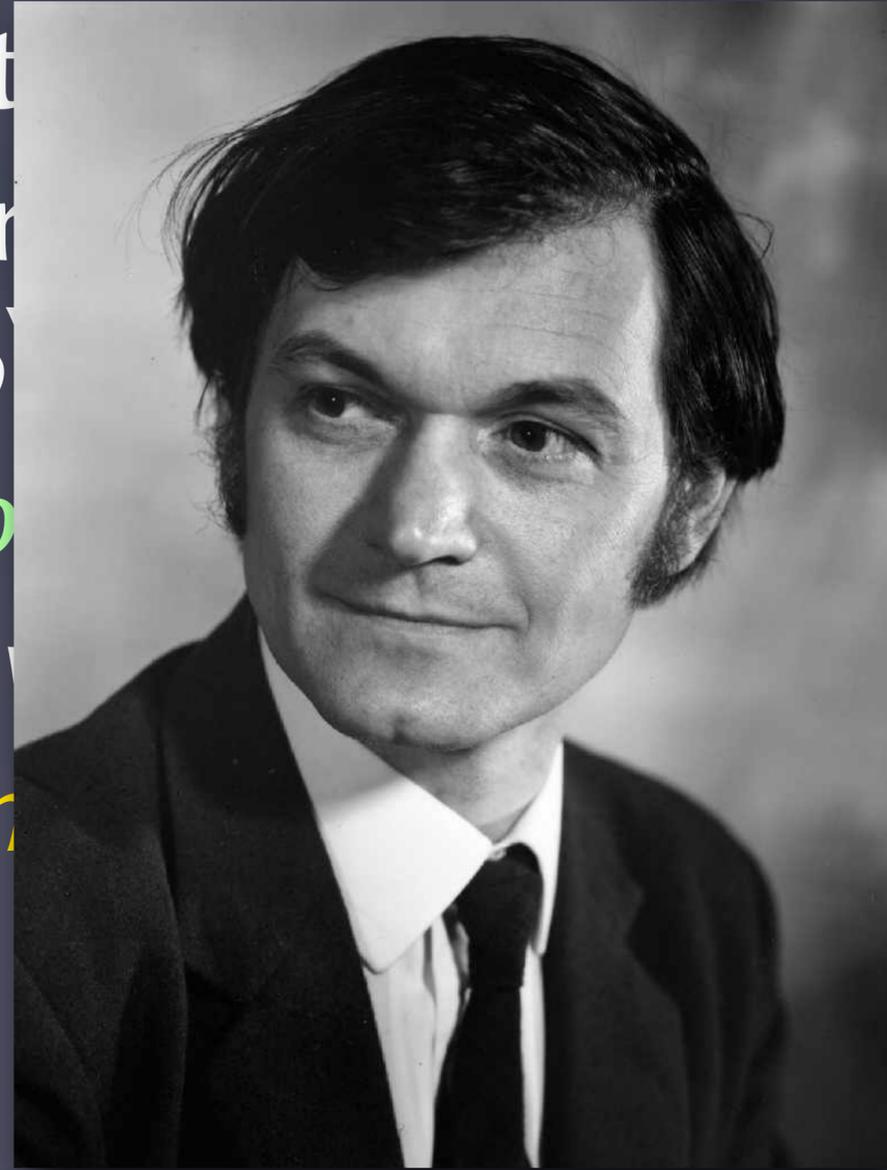


Singularity Theorems



Penrose

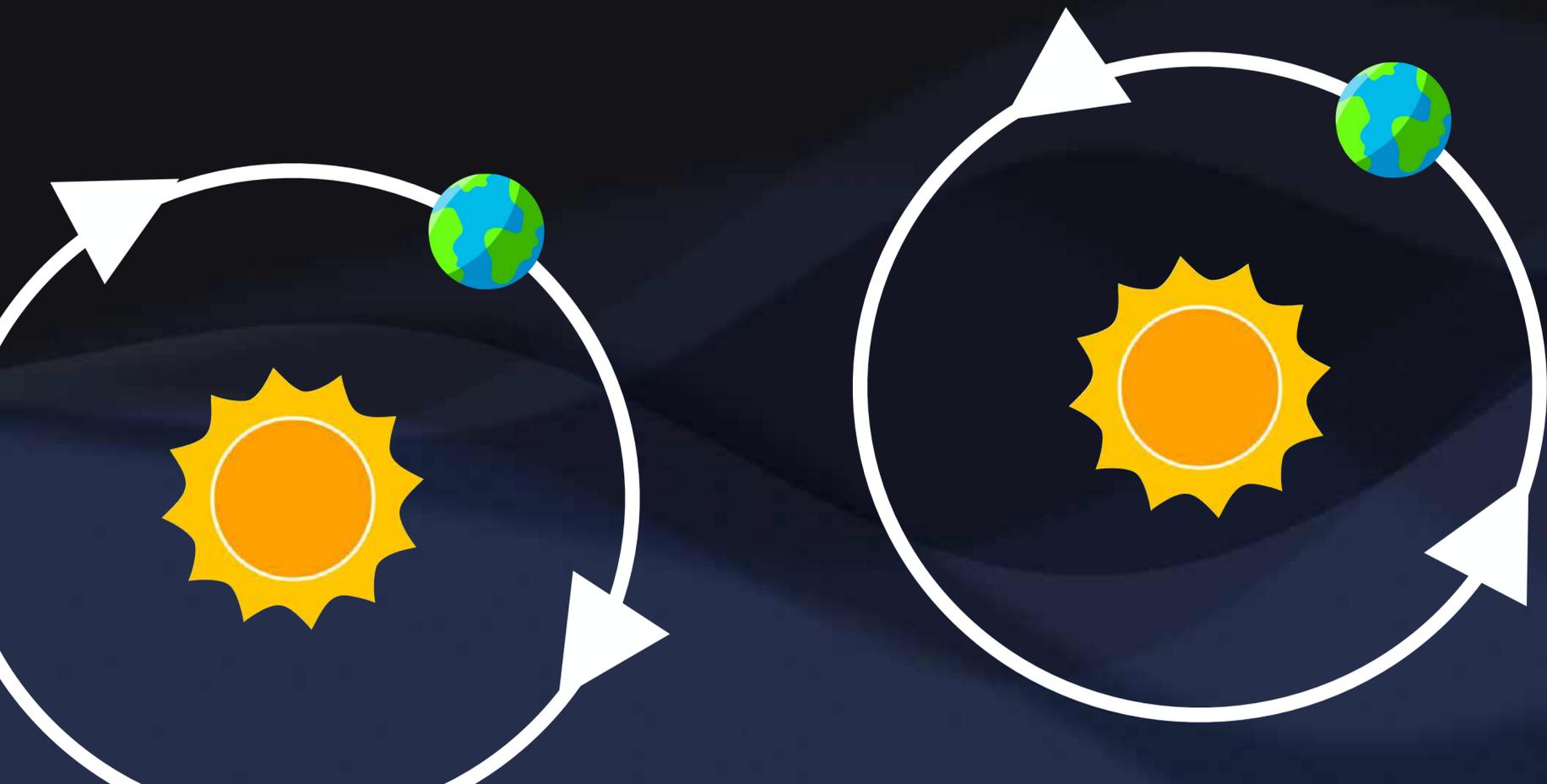
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Time Reversibility

Planets could orbit clockwise, or counterclockwise about the sun.



A ball thrown upwards, or dropped from a great height both follow the same path.



1



3



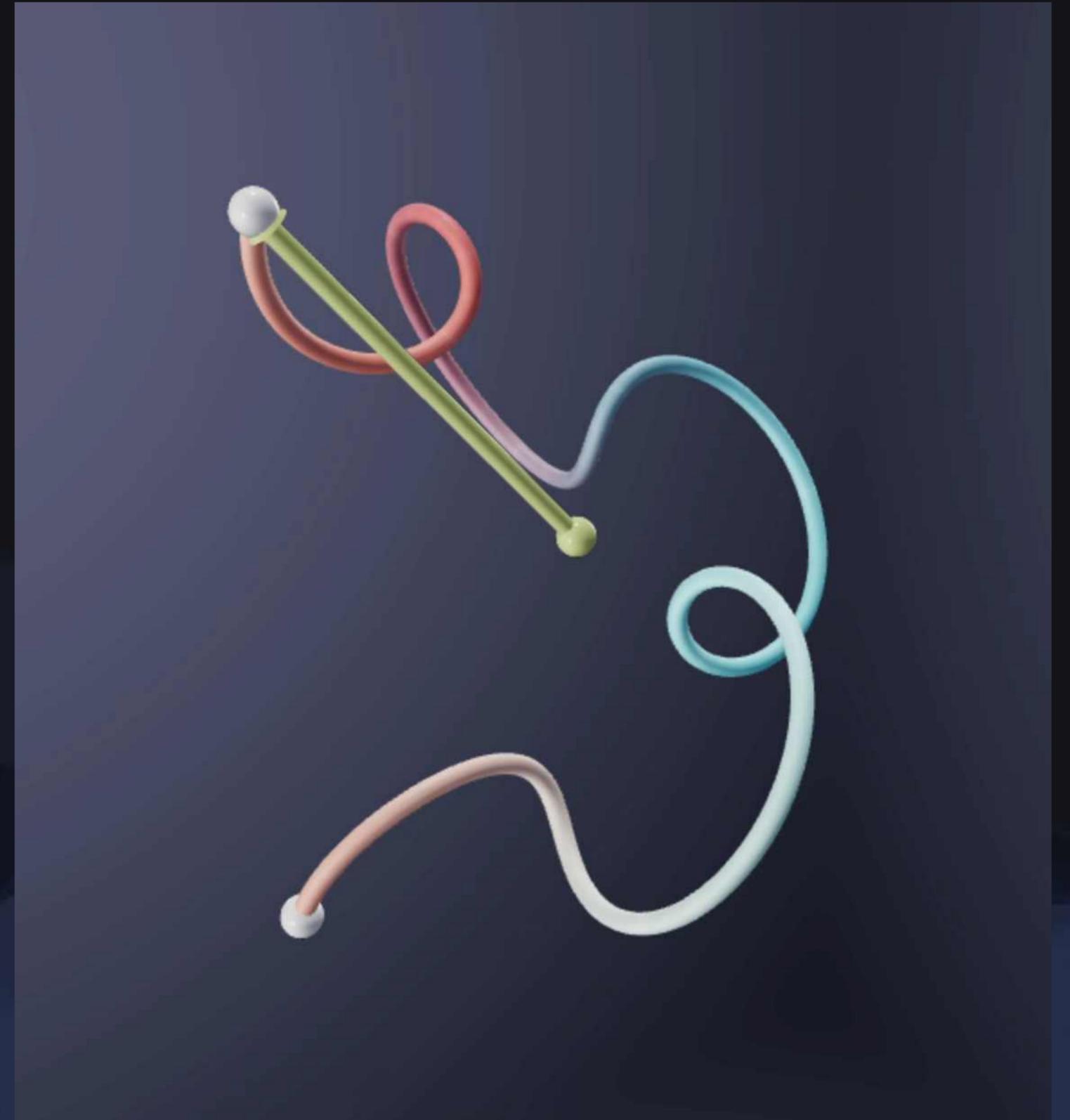
5



Time Reversibility

Mathematical Physics:

*If $c(t)$ is a solution to
Newton's laws of motion,
then $c(-t)$ is also a solution.*



Time Reversibility

Assume:

$$\vec{c}(t)$$

solves

$$m\ddot{c}(t) = F(c(t))$$

Prove:

$$\vec{c}(-t)$$

solves

$$m\ddot{c}(-t) = F(c(-t))$$

Time Reversibility

$$c(-t)'' = \frac{d}{dt} \frac{d}{dt} c(-t)$$

$$\frac{d}{dt} c(-t) = -c'(-t)$$

$$\frac{d}{dt} \frac{d}{dt} c(-t) = - \frac{d}{dt} c'(-t) = c''(-t)$$

Time Reversibility

$$c(-t)'' = c''(-t)$$

$$mc(-t)'' = \underbrace{mc''(-t)}$$

We know this equals

$$F(c(-t))$$

Time Reversibility

$$m c(-t)'' = F(c(-t))$$



$c(-t)$ solves Newton's Equation

Time Reversibility

Theorem

$c(t)$ solves Newton's Equation



$c(-t)$ solves Newton's Equation

Time Reversibility



If physics is
reversible,
why does
time appear
to go in only
one direction?



PART II

PREDICTABILITY

Predictability

We use physics to make predictions about the future.

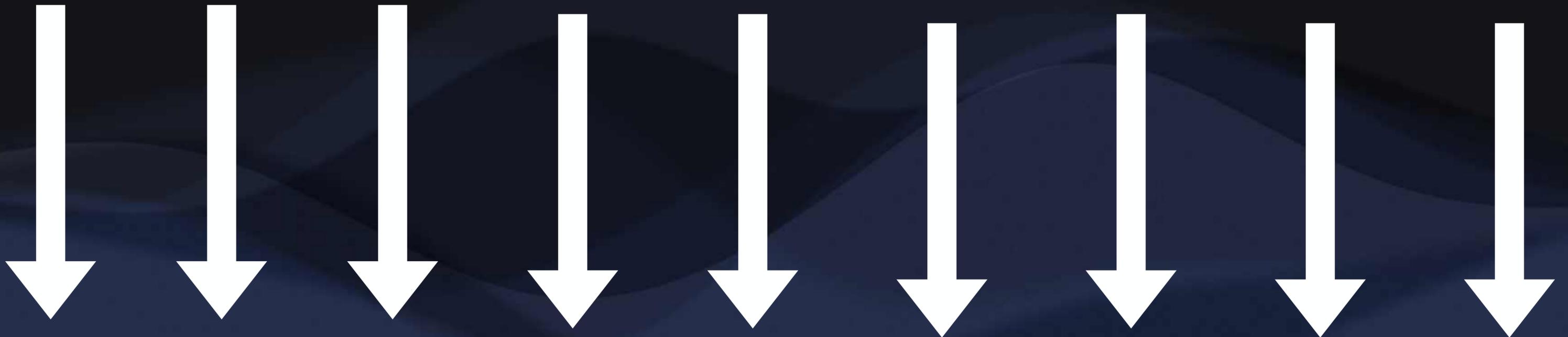
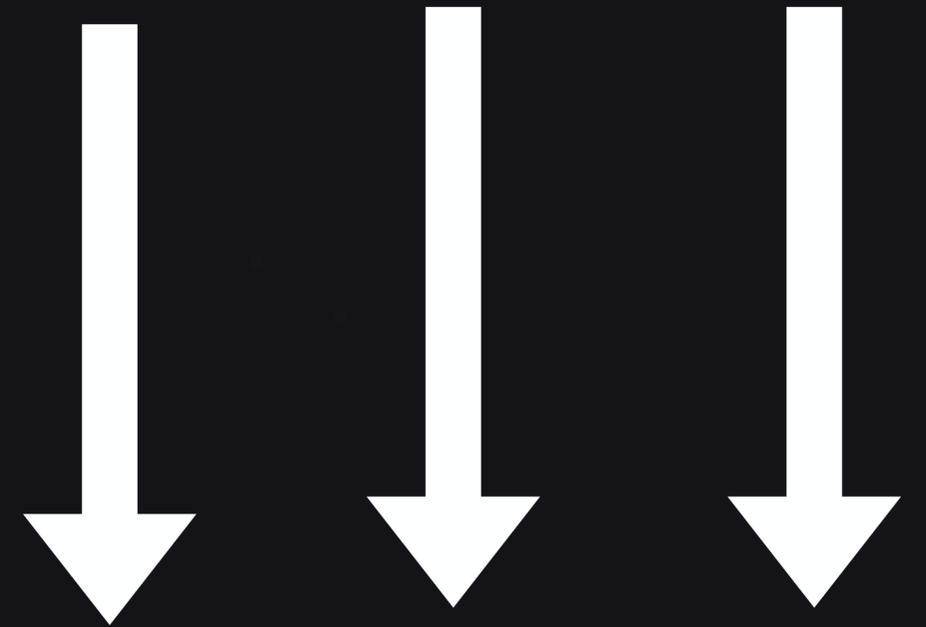
Is this justified?

Mathematical Physics:
Do the **laws** of **Newtonian mechanics** uniquely predict the future?



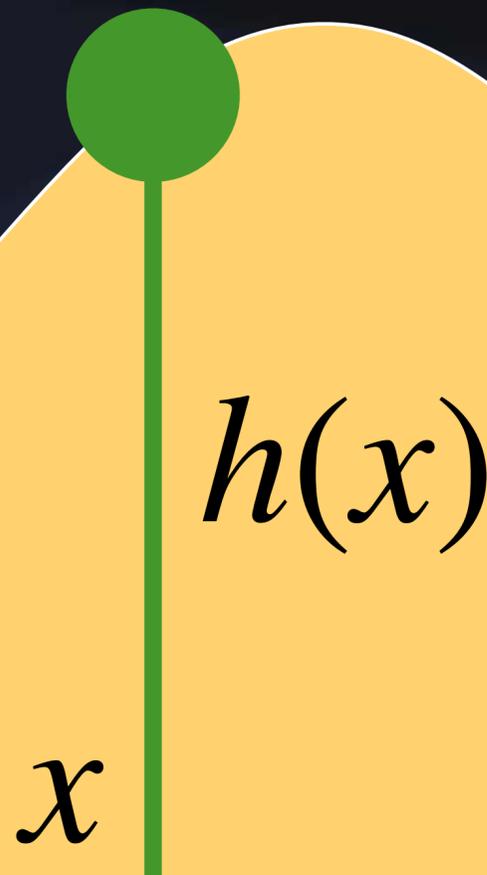
GALILEAN GRAVITY

Physics in a uniform gravity field.



GALILEAN GRAVITY

Potential: $mg h(x)$



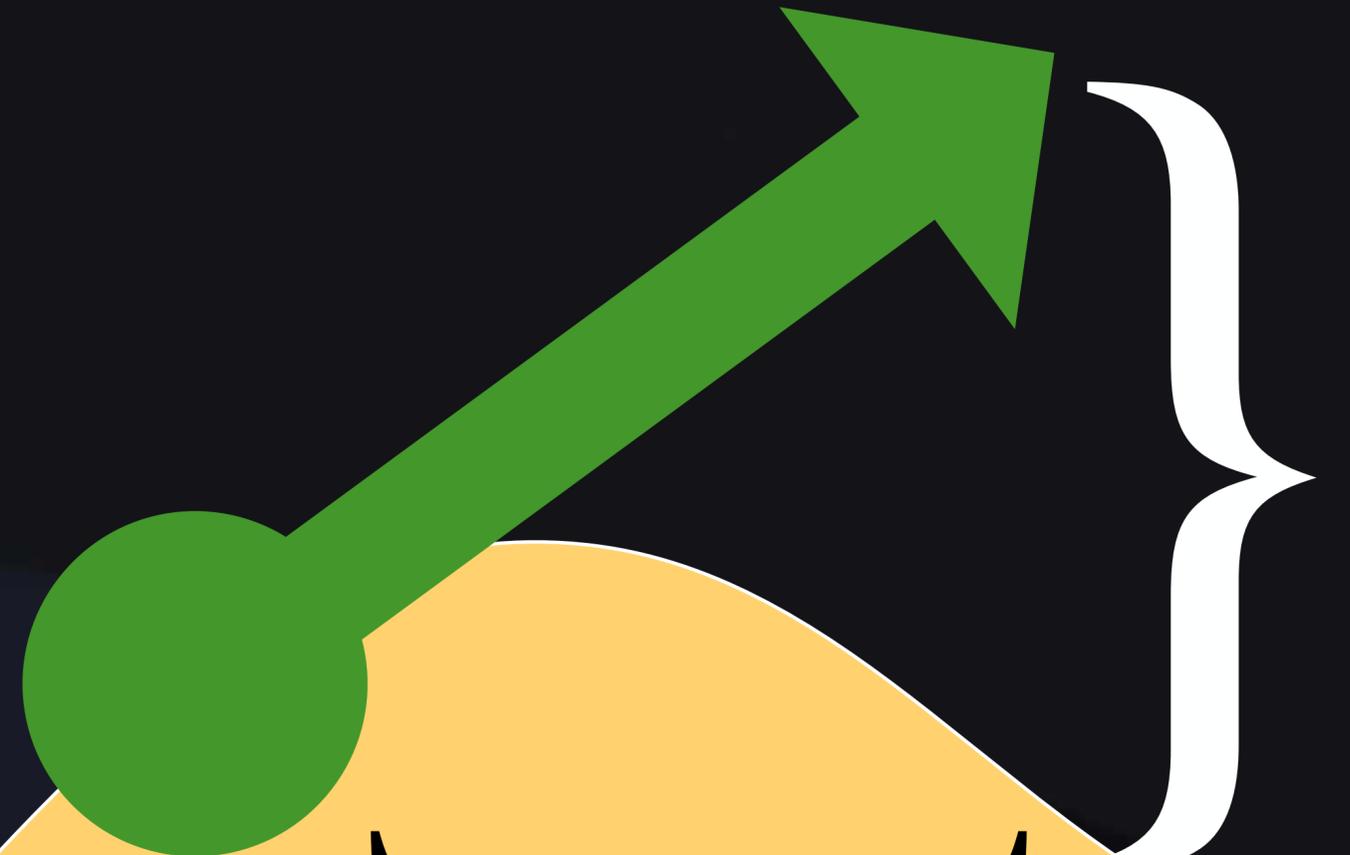
GALILEAN GRAVITY

Potential: $mg h(x)$

Kinetic: $\frac{1}{2}mv^2$

$$\frac{1}{2}m(\dot{x}^2 + h'(x)^2 \dot{x}^2)$$

$$h'(h'(x)\dot{x})'$$



x'

GALILEAN GRAVITY

$$\ddot{x} = -h'(x) \frac{h''(x)\dot{x}^2 + g}{1 + h'(x)^2}$$

Potential: $mg h(x)$

Kinetic: $\frac{1}{2}m(\dot{x}^2 + h'(x)^2\dot{x}^2)$

GALILEAN GRAVITY

$$\ddot{x} = -h'(x) \frac{h''(x)\dot{x}^2 + g}{1 + h'(x)^2}$$

Solving this equation tells us how a ball will move on this hill.

GALILEAN GRAVITY

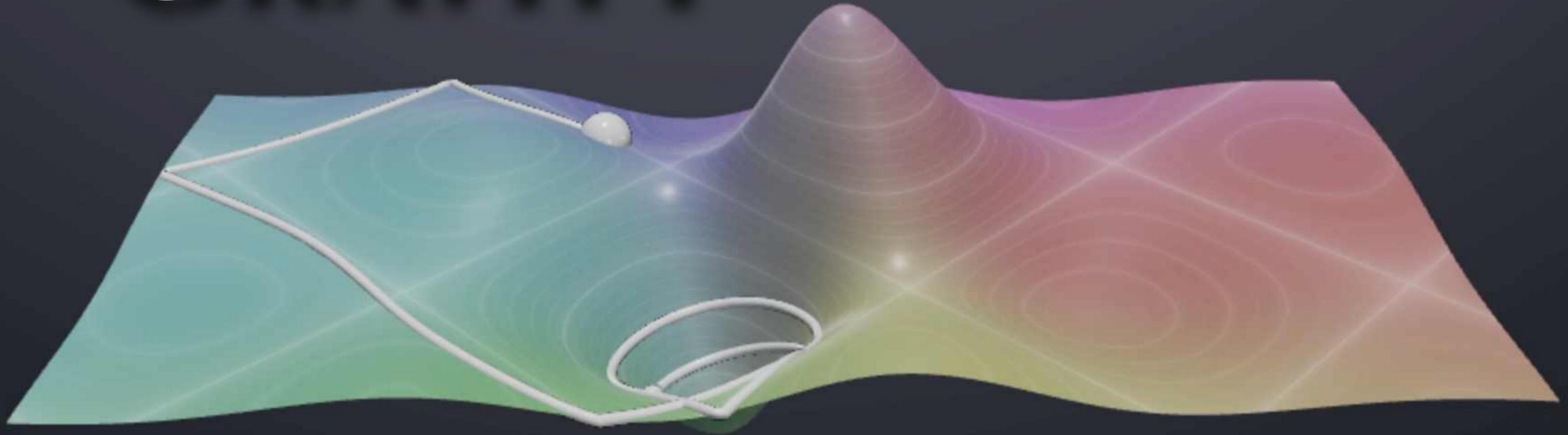
In 3 Dimensions:

$$(1 + h_x^2 + h_y^2) \ddot{x} = - (h_{xx} \dot{x}^2 + 2h_{xy} \dot{x}\dot{y} + h_{yy} \dot{y}^2 + g) h_x$$

$$(1 + h_x^2 + h_y^2) \ddot{y} = - (h_{xx} \dot{x}^2 + 2h_{xy} \dot{x}\dot{y} + h_{yy} \dot{y}^2 + g) h_y$$

Solving this equation tells us how a ball
will move on this hill.

GALILEAN GRAVITY



GALILEAN GRAVITY

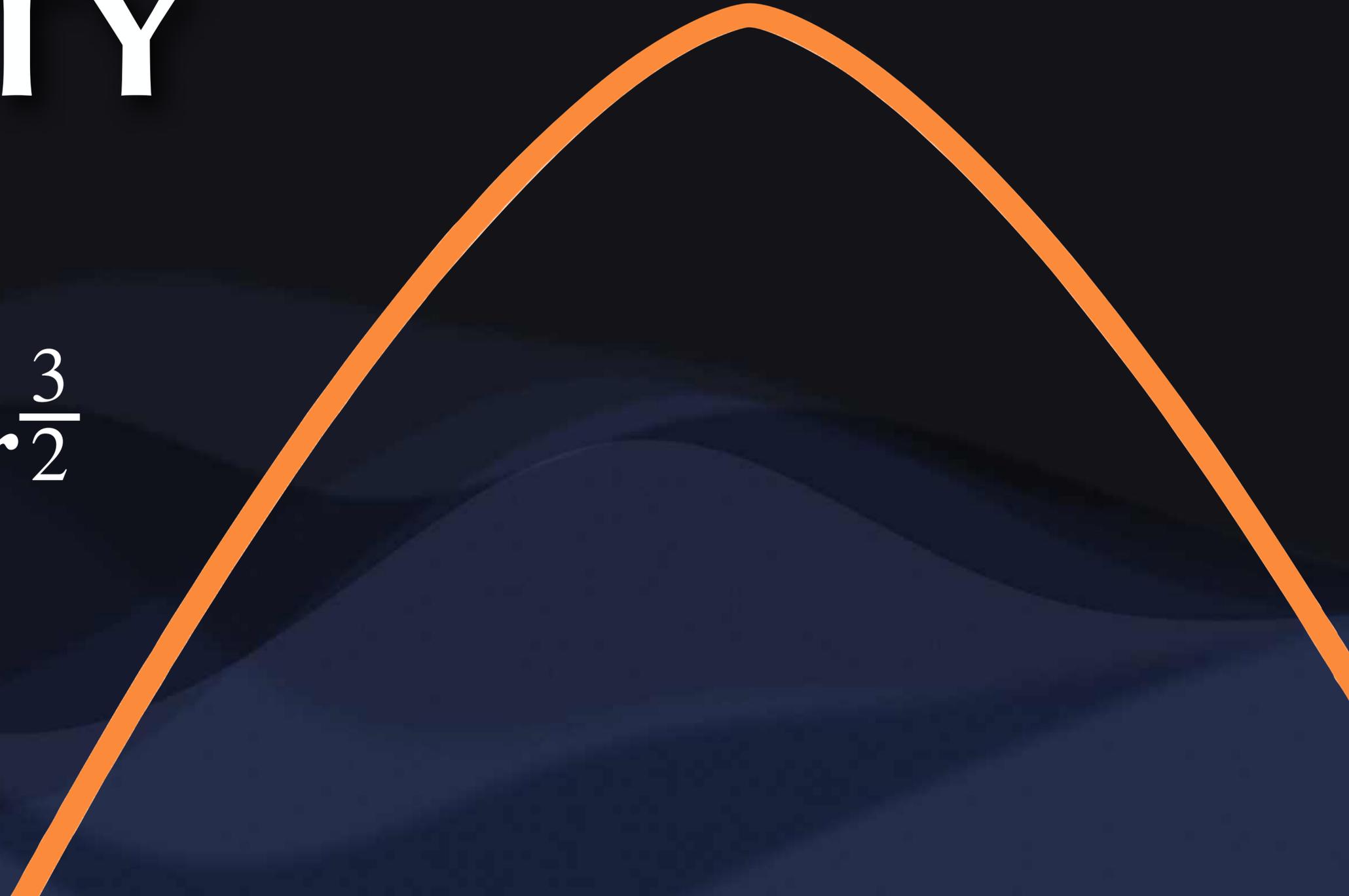


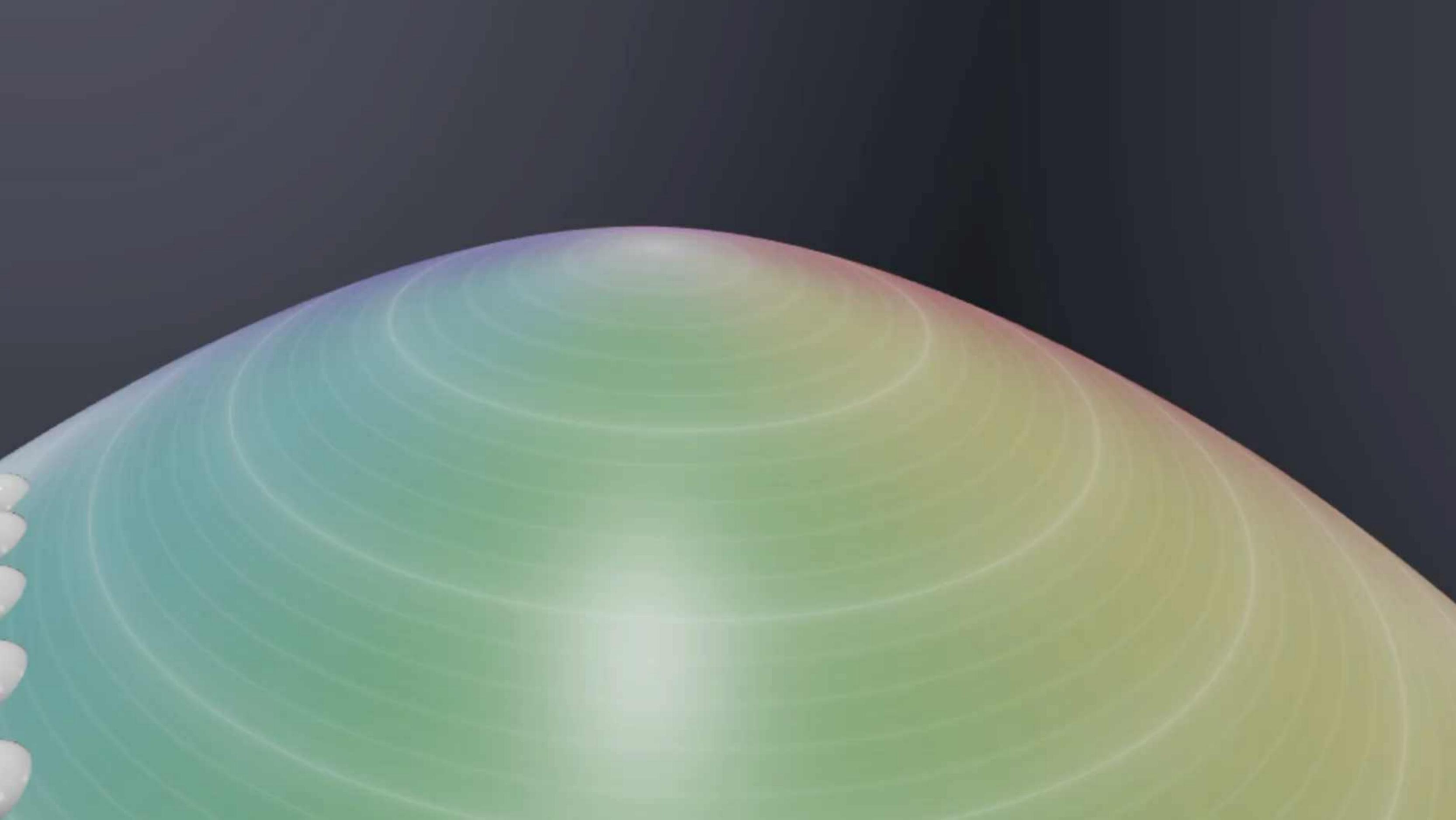
GALILEAN GRAVITY



GALILEAN GRAVITY

$$h(r) = \frac{2}{3g} r^{\frac{3}{2}}$$



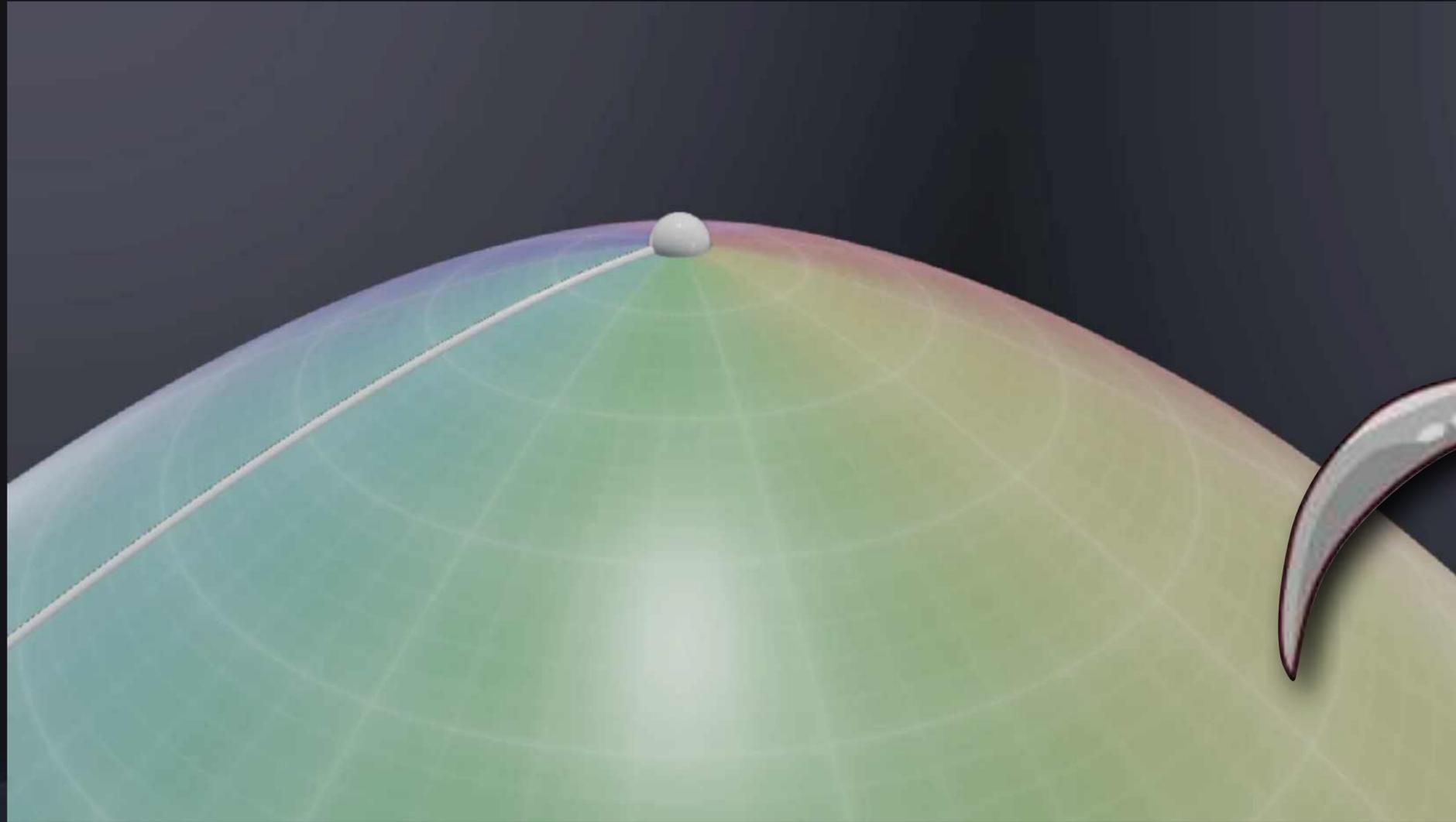


WHY IS THIS SO SCARY?

Physics is
symmetric under
time reversal!



WHY IS THIS SO IMPORTANT?



There's a **solution** to Newton's laws where a ball sits atop the vertex, and then at **some random time** (and in **some random direction**) starts to roll!

GALILEAN GRAVITY

Confirm:

$$r'' = \sqrt{|r|}$$

$$r(t) = \frac{1}{144}(t - T)^4$$

$$h(r) = \frac{2}{3g}r^{\frac{3}{2}}$$

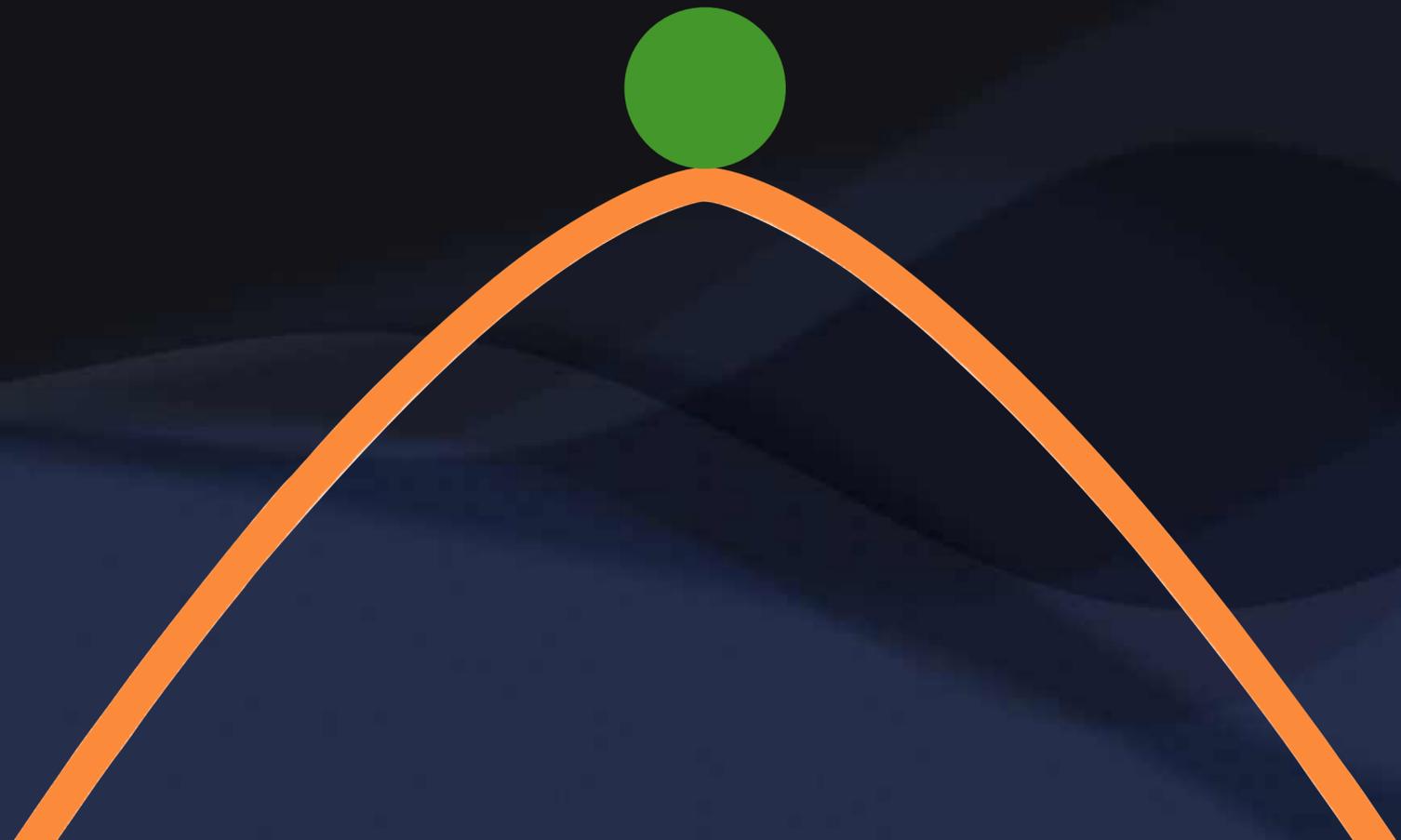
T is **ANYTHING!**

GALILEAN GRAVITY

**Newtonian physics
does not uniquely
predict the future of
this system.**



GALILEAN GRAVITY



Newtonian physics
IS NOT
a predictive theory!

Example of Indeterminacy in Classical Dynamics

Sanjay P. Bhat¹ and Dennis S. Bernstein¹

Received May 13, 1996

The case of a particle moving along a nonsmooth constraint under the action of uniform gravity is presented as an example of indeterminacy in a classical situation. The indeterminacy arises from certain initial conditions having nonunique solutions and is due to the failure of the Lipschitz condition at the corresponding points in the phase space of the equation of motion.

1. INTRODUCTION

An often unstated assumption of classical mechanics is that the laws of dynamics yield deterministic models. This assumption is formally captured

**I don't like this example:
it relies on the special
shape of the dome!**



**Alright, follow
me...**



John D. Norton

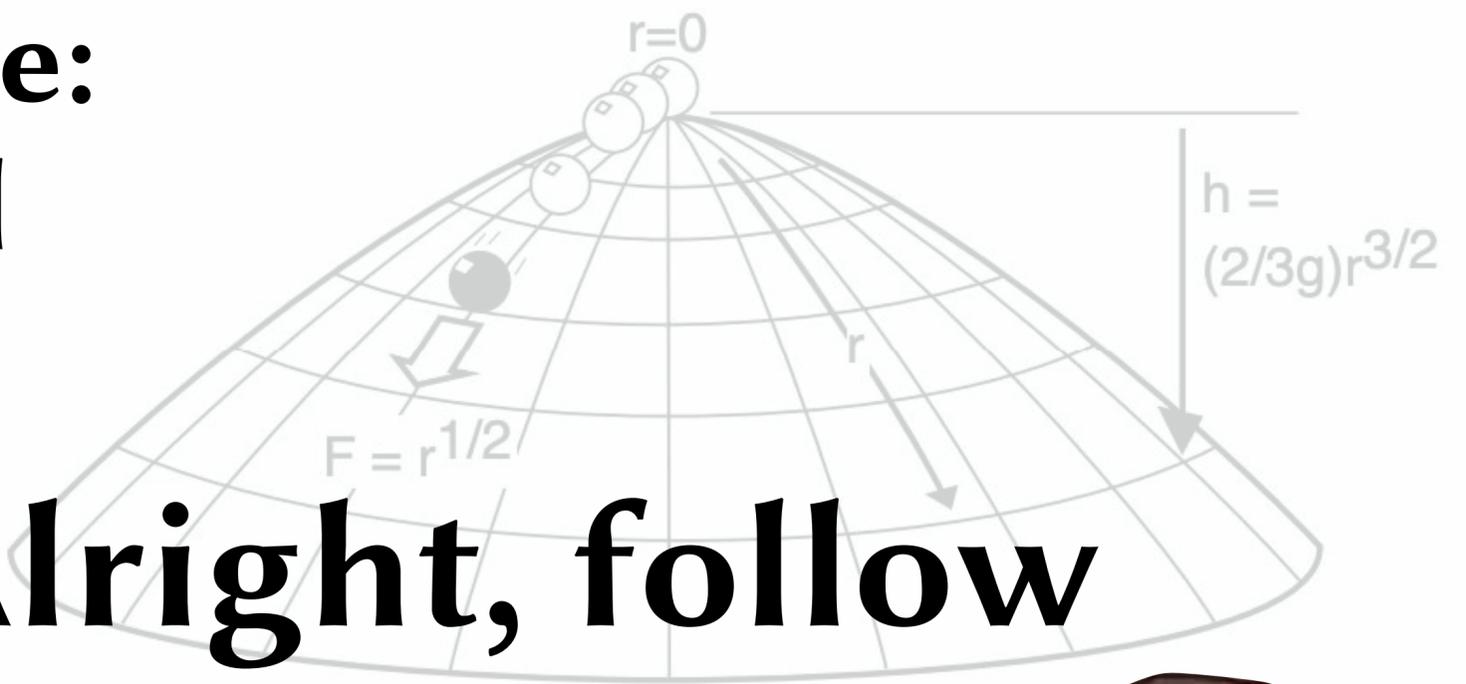


Figure 1a. Mass sliding on a dome

A point-like unit mass slides under the influence of gravity along the dome. The magnitude of the gravitational force is $F = d(gh)/dr = r^{1/2}$ and there is no tangential force at $r=0$. The mass experiences a net outward force of magnitude $r^{1/2}$. Newton's second law applied to the mass on the surface, sets the radial acceleration equal to the magnitude of the force field:

$$(1) \quad d^2r/dt^2 = r^{1/2}$$

If the mass is initially located at $r=0$, there is one

NEWTONIAN GRAVITY

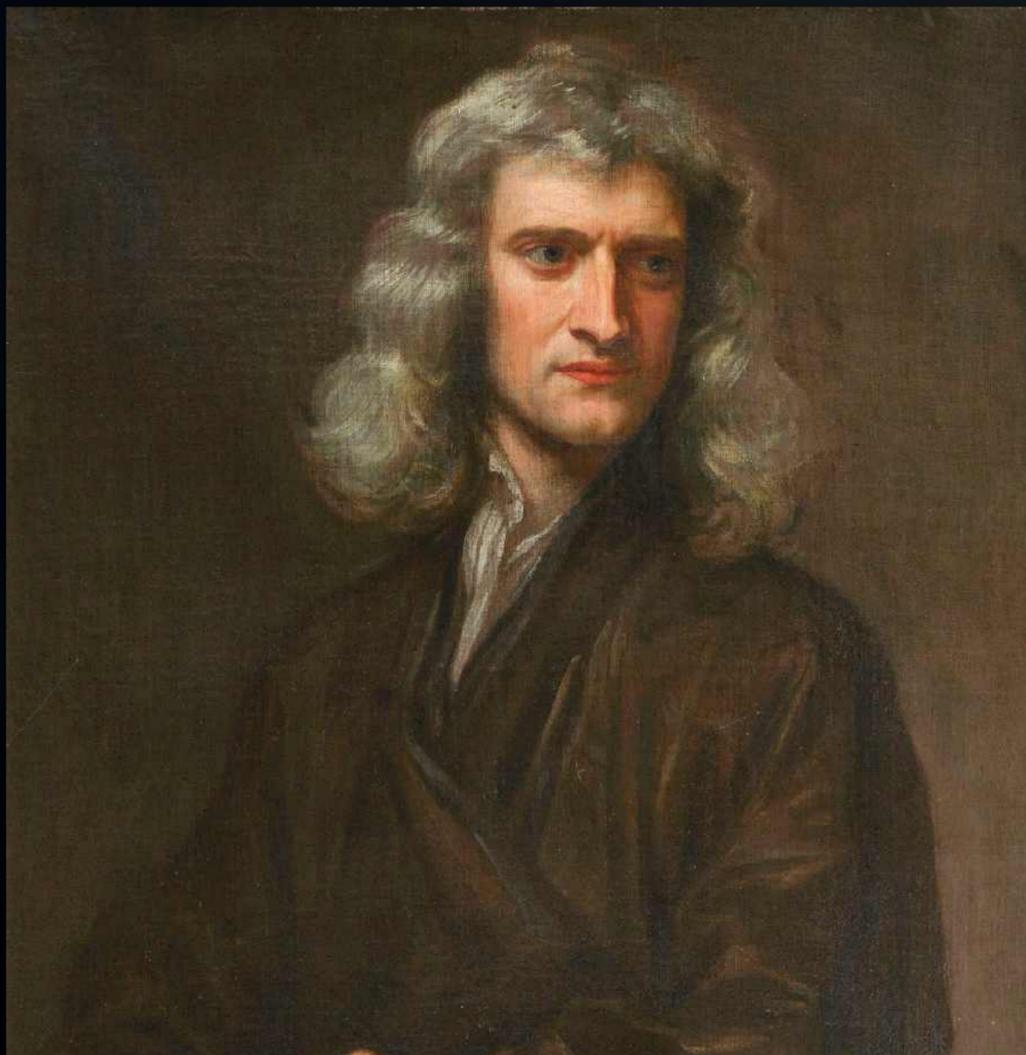
2 Bodies orbit
on ellipses, for
all time...



NEWTONIAN GRAVITY

.....except if
they collide.

*We call solutions that can't be
continued into the future
singularities.*



Theorem (Newton):

The only **singularities**
of the 2-body problem
are **collisions.**

NEWTONIAN GRAVITY

What about when
there are three or
more bodies?



Theorem (Newton):

The two-body problem
either has a unique
solution for all time, or
the two bodies collide.

NEWTONIAN GRAVITY



King Oskar

1895: Prize for
mathematical progress
on the n-body problem
of Newtonian
gravitation



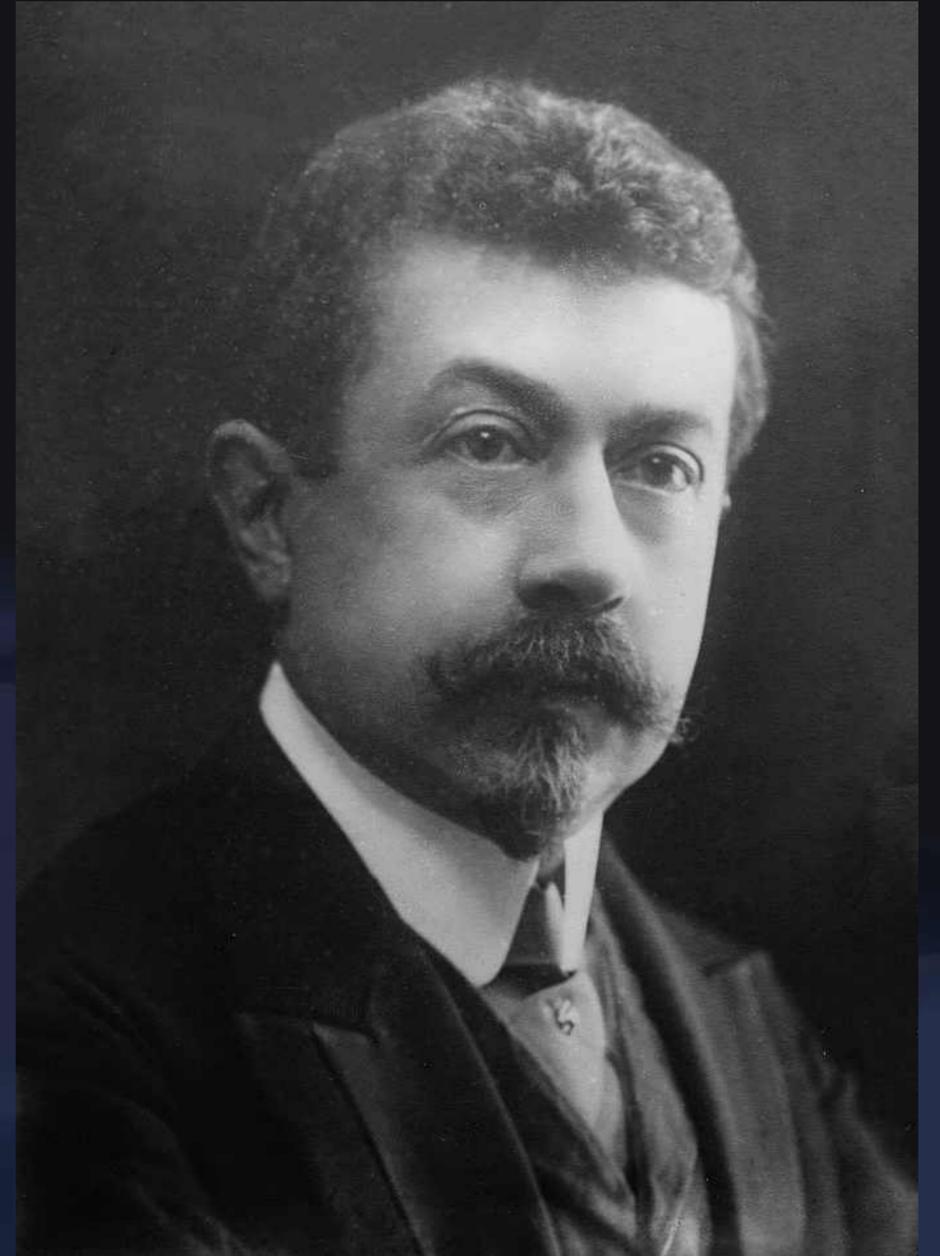
NEWTONIAN GRAVITY

Painlevé



King Oskar

1895: Prize for
mathematical progress
on the n-body problem
of Newtonian
gravitation



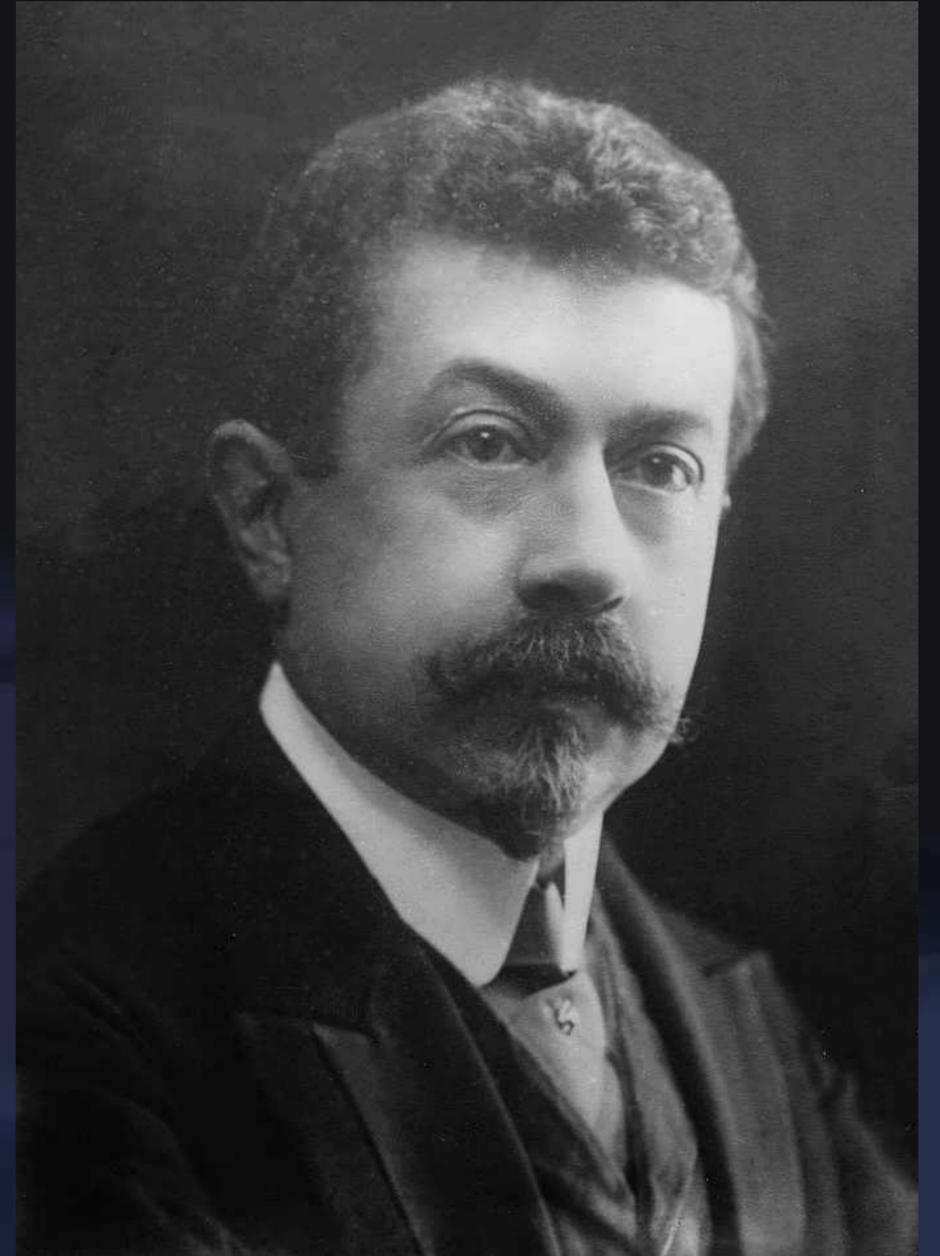
NEWTONIAN GRAVITY

*Twice the prime
minister of France!*

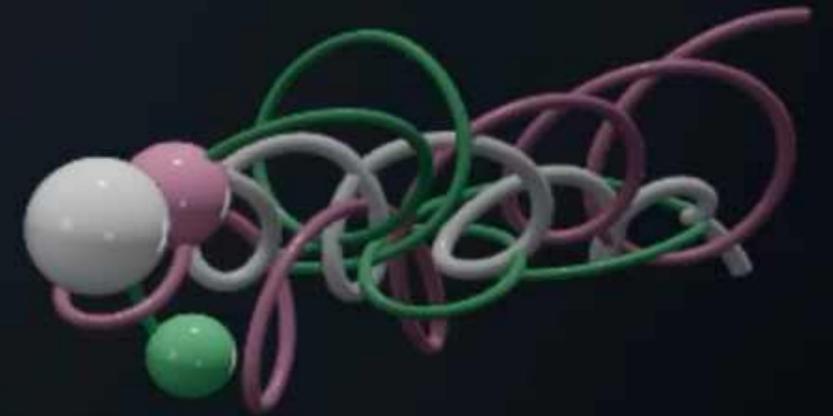
Theorem (Painlevé):

The only **singularities** of the 3-
body problem are **collisions**.

Painlevé

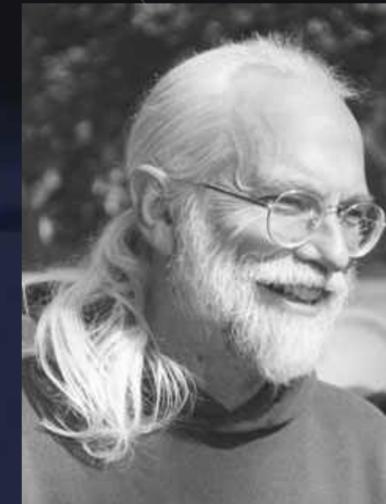
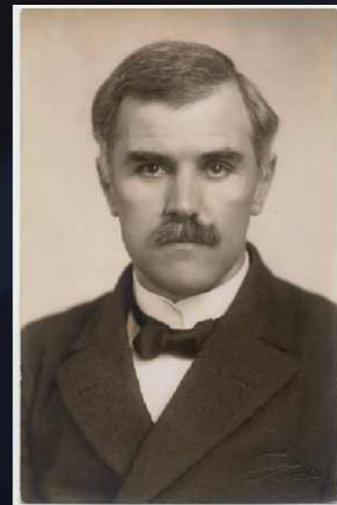
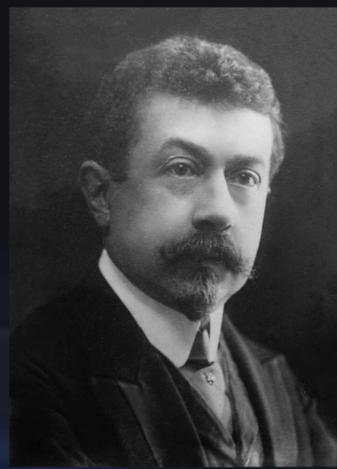


NEWTONIAN GRAVITY



NEWTONIAN GRAVITY

What about when there
are FOUR or more bodies?



1895

Poincare, Panlieve

1908

von Zeipel

1975

Mather, McGeehee

...



The existence of noncollision singularities in newtonian systems

By ZHIHONG XIA*

Introduction

In this paper we solve a long-standing problem in celestial mechanics proposed by Painlevé and Poincaré in the last century. The problem, which concerns the nature of the singularities in the n -body problem, asks whether there exists a noncollision singularity in the newtonian n -body problem? Here we give an affirmative answer to this problem by proving the existence of noncollision singularities in the 5-body problem.

We consider n point-masses moving in a euclidean space \mathbb{R}^3 . Let the mass of the i^{th} particle be $m_i > 0$, let its position be $\mathbf{q}_i \in \mathbb{R}^3$ and let $\dot{\mathbf{q}}_i \in \mathbb{R}^3$ be its velocity. According to Newton's law,

$$(0.1) \quad m_i \ddot{\mathbf{q}}_i = \sum_{j \neq i} \frac{m_i m_j}{|\mathbf{q}_i - \mathbf{q}_j|^3} (\mathbf{q}_i - \mathbf{q}_j) = \frac{\partial U}{\partial \mathbf{q}_i},$$

1988
Jeff Xia



XIA'S SINGULARITY

3-Body
Oscillation



XIA'S SINGULARITY

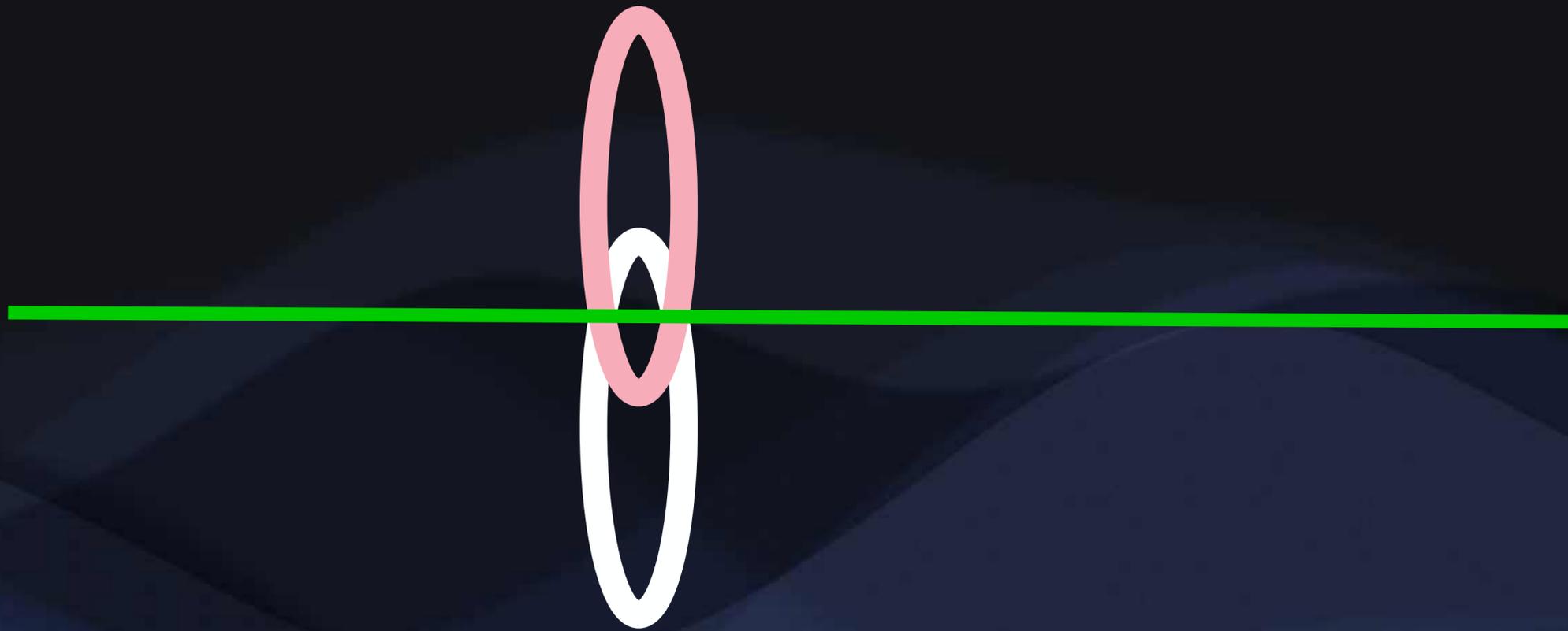
3-Body
Accelerator

Time it
perfectly
so the
third mass
picks up
downward
speed.



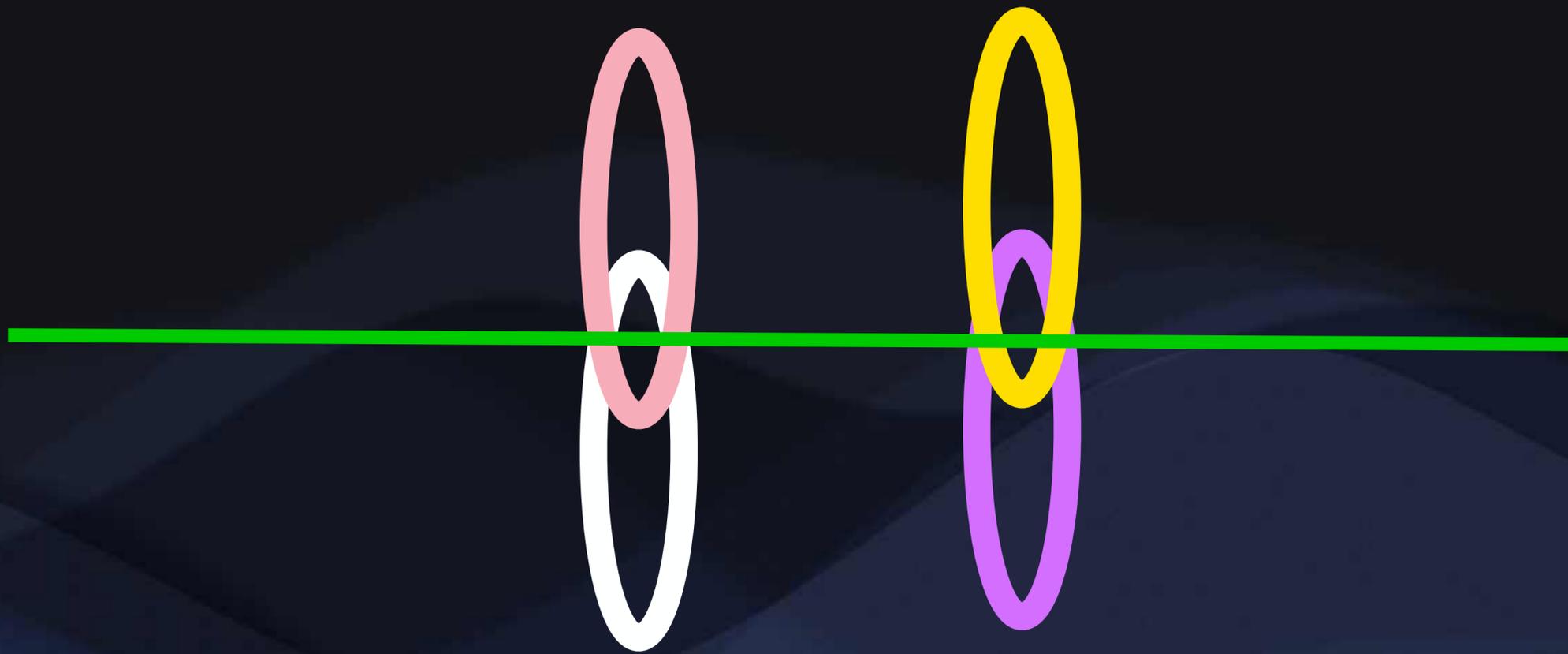
XIA'S SINGULARITY

One
Accelerator



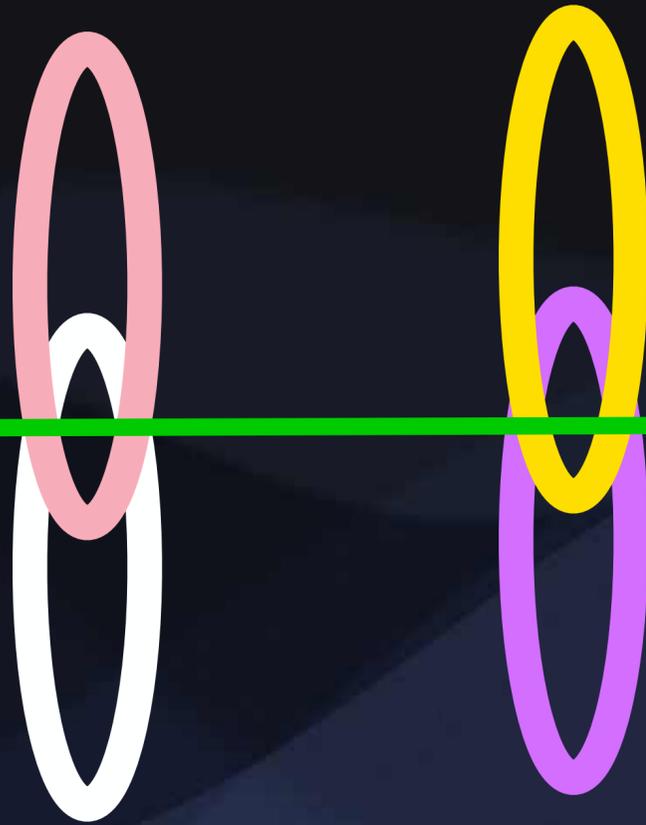
XIA'S SINGULARITY

Two
Accelerators



XIA'S SINGULARITY

Two
Accelerators
**That are also
oscillators!**



XIA'S SINGULARITY

Two
Accelerators
**That are also
oscillators!**



XIA'S SINGULARITY

Two
Accelerators
**That are also
oscillators!**



XIA'S SINGULARITY

The **white** and **pink** masses go to $-\infty$. The **yellow** and **purple** masses go to $+\infty$. The **green** mass oscillates infinitely often, going to $\pm\infty$

In a finite amount of time.



SPACE INVADERS

If 5 masses can disappear to infinity... what happens when you reverse time?



SPACE INVADERS

If 5 masses can disappear to infinity... what happens when you reverse time?



SPACE INVADERS

Definition:

A **Space Invader solution** to Newtonian Mechanics is one where **new particles rush in from infinity** at some point in time.



SPACE INVADERS

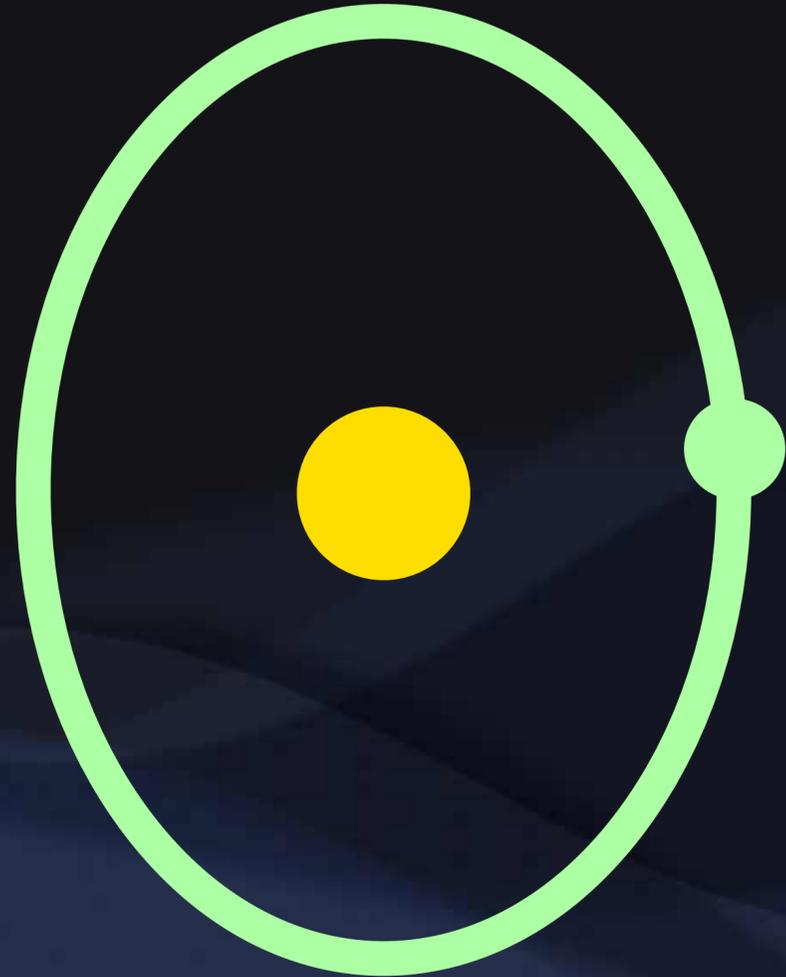
At any moment, you could be swarmed by a multitude of space invaders from infinity!

Mass is not conserved in Newtonian Mechanics!
Number of particles is not conserved.



SPACE INVADERS

Future histories of
the...two body problem of
Newton?!



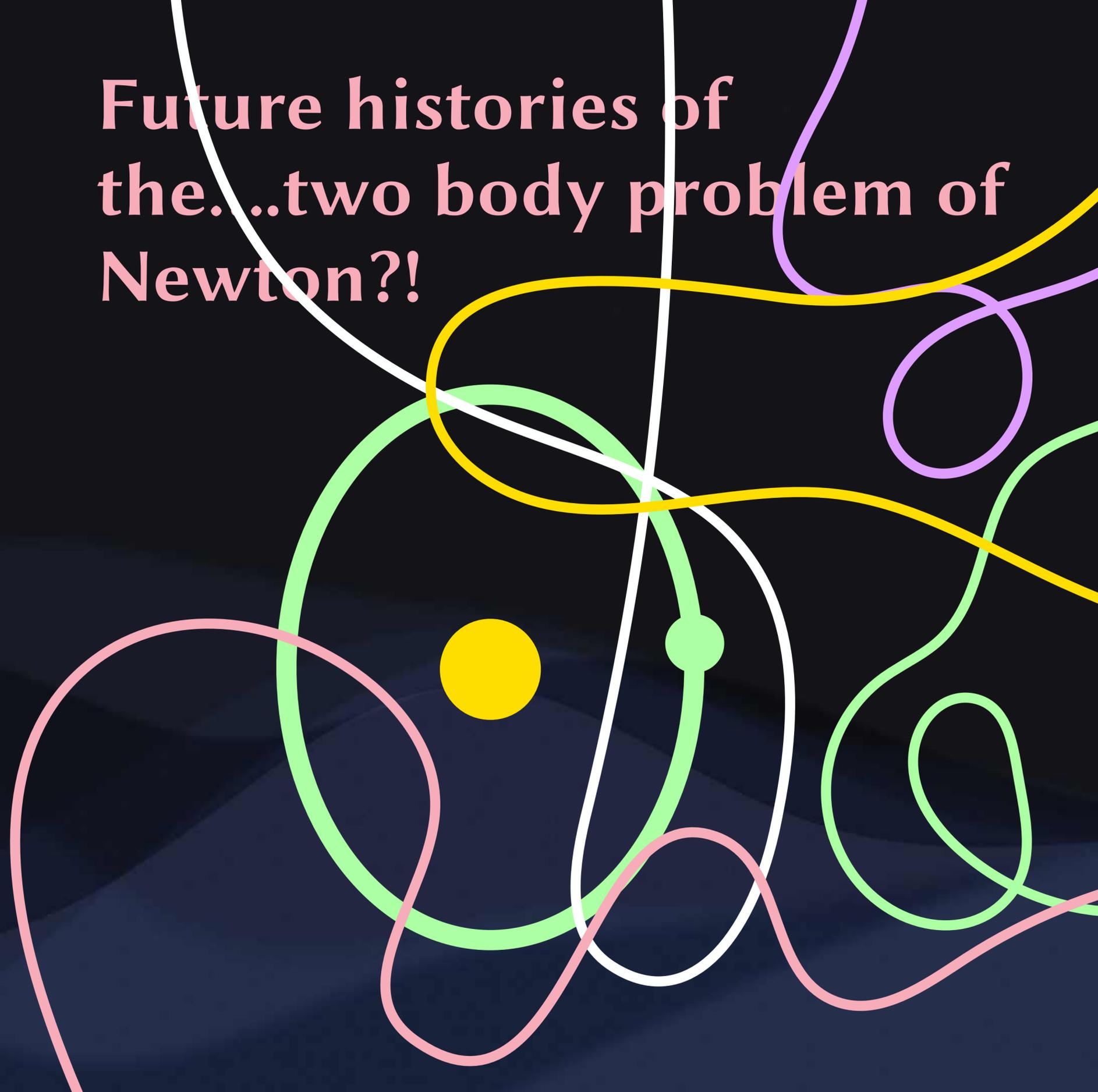
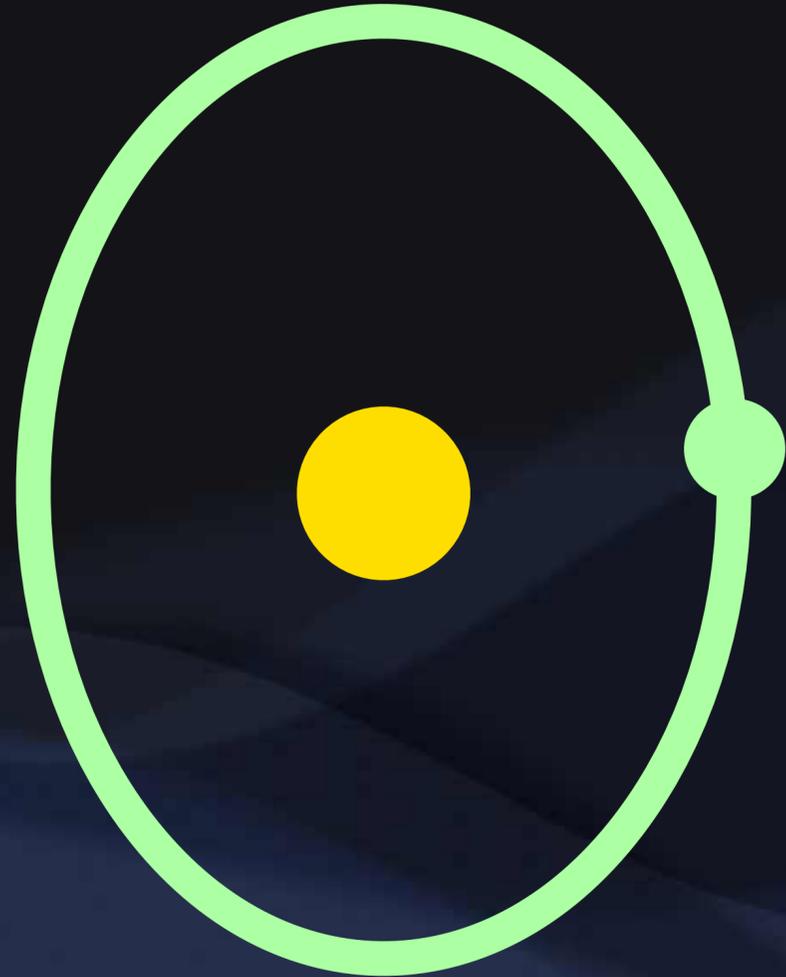
SPACE INVADERS

Future histories of
the...two body problem of
Newton?!



SPACE INVADERS

Future histories of the...two body problem of Newton?!



4 BODY NON-COLLISION

Gerver, Huang, Xue 2022

Xue 2014

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Non-collision singularities in a planar 4-body problem

by

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Beijing, China*

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NEW MECHANISM FOR NONCOLLISION SINGULARITIES

JOSEPH GERVER, GUAN HUANG, JINXIN XUE

ABSTRACT. In this paper, we prove the existence of noncollision singularities in the planar four-body problem with a model different from [19]. In this model, the acceleration can be arbitrarily fast and the masses can be comparable. This work provides a general principle to construct noncollision singularities as well as other related orbits with complicated dynamics. It not only answers a question in [19] but also solves an analogous version of a conjecture of Anosov.

1. INTRODUCTION

In this paper, we prove the existence of noncollision singularities in a model of the four-body problem which is drastically different from the model of [19]. Noncollision singularities are singularities of the N -body problem for which no collision occurs. Their dynamical behaviors are wild. Indeed, when approaching the singular time, the orbit determined by a noncollision singularity has to oscillate infinitely often between smaller and smaller neighborhoods of collision and infinity. It had been a longstanding conjecture that noncollision singularities exist in the Newtonian N -body problem for $N > 3$. The conjecture was finally answered positively after a century [13, 6, 18] etc. In this paper, we use a new acceleration mechanism, which is much faster than the one in [19] and more importantly gives a very general principle to construct noncollision singularities and other orbits with complicated dynamical behaviors. The model in this paper does not rely on small perturbations of Kepler motions so it works for comparable masses, including in particular four equal masses.



The Reaper MARCHES ON

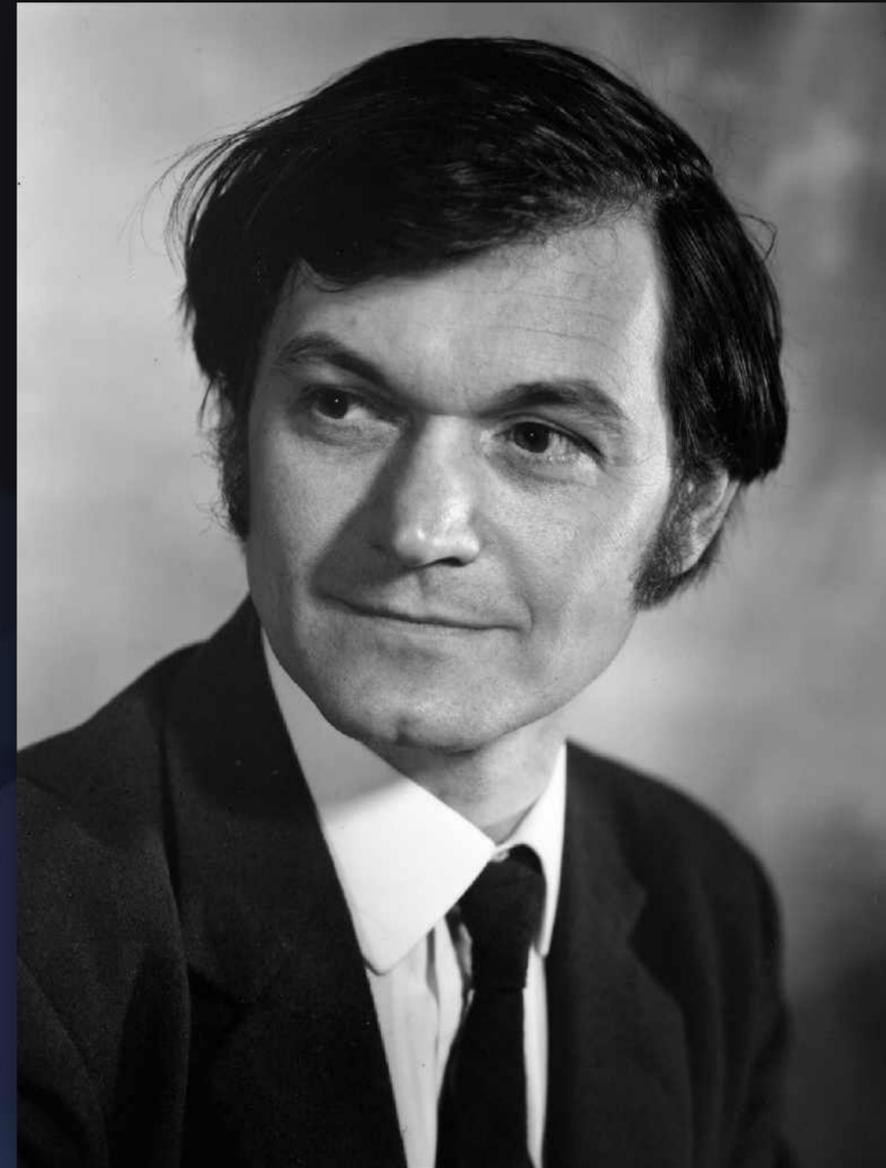


General Relativity

1969
Penrose

**“Strong Cosmic
Censorship”**

General relativity is a
deterministic theory.



General rela

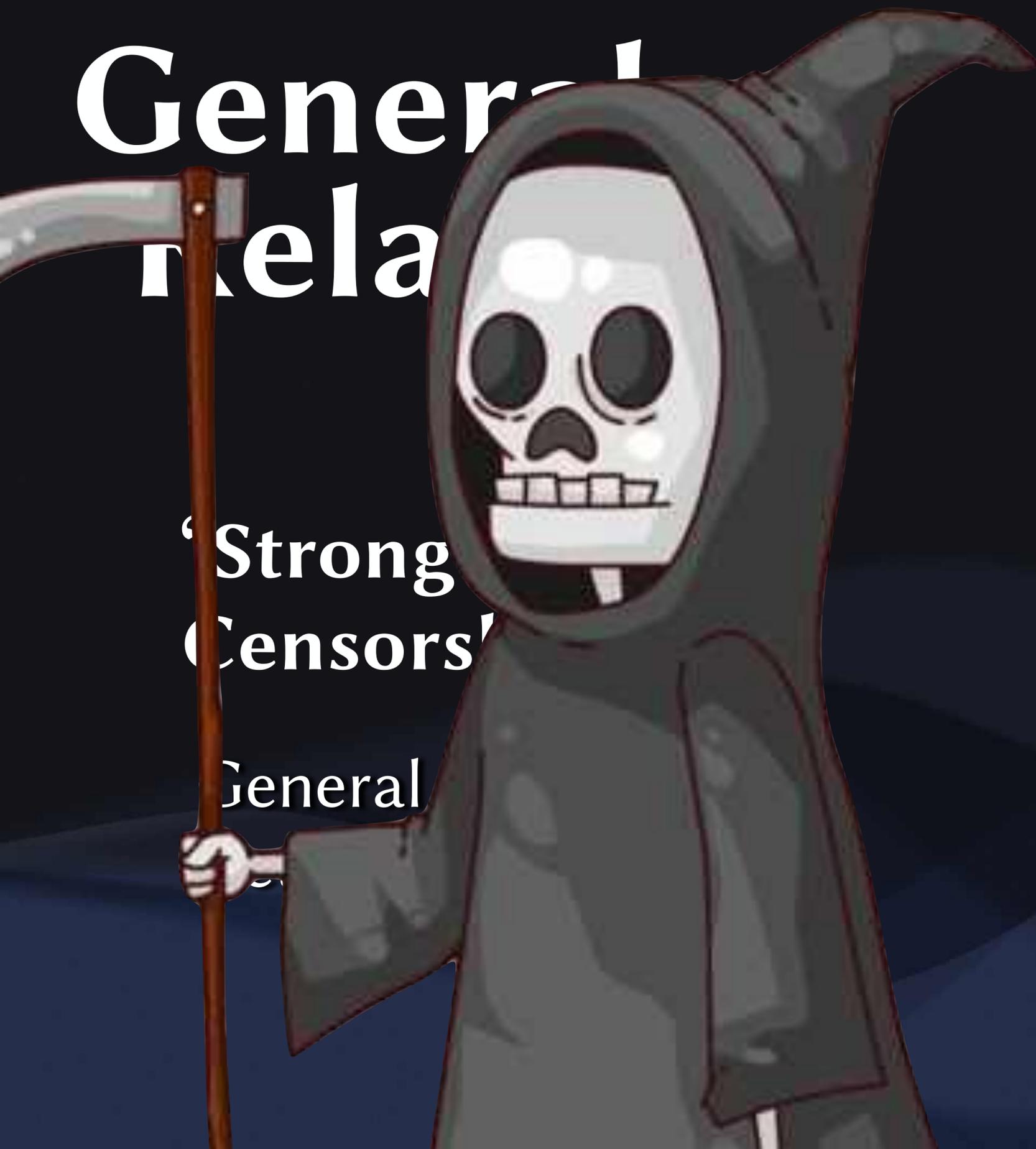
2018

Luk

Dafermos

'Strong
Censors'

General



Fluid Mechanics

[Home](#) — [Millennium Problems](#) — Navier-Stokes Equation

\$1,000,000

Unsolved

Navier-Stokes Equation

This is the equation which governs the flow of fluids such as water and air. However, there is no proof for the most basic questions one can ask: do solutions exist, and are they unique? Why ask for a proof? Because a proof gives not only certitude, but also understanding.

